Heterogeneous Beliefs and FOMC Announcements[*]

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Abstract

This paper studies the effect of FOMC announcements on the dynamics of heterogeneous beliefs. The open interest of options decreases significantly after announcements, implying the associated high trading volume comes from unwinding positions with less disagreement. To measure this effect, I develop a quantitative general equilibrium model with heterogeneous beliefs about fundamental growth under recursive utility. On average, the disagreement on growth decreases from 1.15% to 0.55% on announcement days for recent years. In a counterfactual economy where the transparency of announcements decreases by half, disagreement increases 0.31%, and stock market volatility is 16.8% higher due to more speculative trading.

Keywords: Heterogeneous beliefs; FOMC announcement; trading volume

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1 Introduction

Heterogeneous beliefs play an essential role in linking asset prices and economic fluctuations. While the arrival of news is the primary driver of changing beliefs, very little attention has been paid to how Federal Open Market Committee (FOMC) announcements affect the dynamics of heterogeneous beliefs. A question that naturally arises is whether there is more or less disagreement after macroeconomic information releases. Furthermore, how strongly do the announcements affect heterogeneous beliefs both in the short run and in the long run? How much does this contribute to asset market fluctuations? I aim to answer these questions in this paper.

Empirically, the average 5-minute trading volume of E-mini S&P 500 Futures (E-mini) increases by 3.6 times upon FOMC announcements and keeps at a very high level until 1.5 hours later, which indicates significant dynamics of heterogeneous beliefs. There are two possible explanations for this empirical evidence. One explanation is that investors interpret the public news differently so that their disagreement increases after announcements, which leads to more aggressive trading in their previous positions. The second explanation is that investors disagree less since they update the beliefs from observing the same public news. The need for rebalancing and unwinding their previous positions contributes to the huge trading volume. The two opposite explanations have different signs of beliefs' dynamics, which would lead to different policy implications of FOMC announcements. The key challenge in answering this question is that not only has the use of survey data been criticized on many fronts,[1] but also there is no high-frequency survey data available before and after announcements.

To overcome this challenge, I document, since 2011, the call (put) open interest of S&P 500 index options reduces -53.5% (-47.1%) at the end of the announcement day, which is statistically significant at the 1% level. This suggests, investors take bets on the macroeconomic information before announcements and hold the corresponding positions. After information releases, investors unwind their positions and rebalance their portfolios due to less disagree-

[1] As pointed by Giglio, Maggiori, Stroebel, and Utkus (2019) and Nagel (2019), the survey data has been criticized from the following aspects: (1) survey data is often based on small and unrepresentative samples; (2) it has measurement errors; (3) the respondent may not act following the stated belief; (4) the questions in the survey may not be informative for models.

ment. The change of open interest provides asset-market-based evidence to capture the sign of disagreement dynamics upon announcements, which also sheds light on the underlying model to understand the above features of the financial markets.

To measure the dynamics of beliefs upon FOMC announcements, I develop a general equilibrium model with two types of investors—optimists and pessimists who differ in their beliefs about the long-run mean of aggregate output growth rate. The dynamics of disagreement are endogenously determined, which do not vanish in the long run. On non-announcement days, investors update their beliefs from observing the aggregate output. While the information from aggregate output reduces the disagreement, different long-run means of growth rate lead to higher divergence in investors' opinions. The FOMC announcements carry additional information on the growth rate, which results in less disagreement after information releases. The disagreement dynamics contribute to time-varying speculative trading based on the underlying fundamentals, which generates endogenous fluctuations in asset holdings as well as wealth accumulation. These, in turn, result in excessive stock market volatility and a time-varying risk premium despite the smooth aggregate fundamentals.

In addition to all features in the standard general equilibrium models with heterogeneous beliefs, my model has unique implications on asset markets upon announcements. The degree of speculation among investors reaches peak level just before announcements because of the highest disagreement of underlying fundamentals. They take bets on the incoming information, and the optimists tend to hold more aggressive positions than the pessimists. Upon announcements, the reduction of disagreement results in the huge trading volume since investors unwind their positions. The immediate reallocation of asset holdings indicates the information effect on the marginal propensity to take risk, which has a significant impact on investors' long-run wealth accumulation. The benchmark model implies the average disagreement on growth decreases from 1.15% to 0.55% on announcement days.

The change of disagreement upon announcements also leads to asset price fluctuations, which contributes to the dynamics of SDF under objective measure through three channels. The first channel is the information effect through the variation of the continuation utility in the SDF. Though aggregate consumption does not respond to announcements, the instantaneous reallocation of consumption among investors changes the SDF, which is the second channel. The third channel comes from the reduction of belief deviations relative to objective measure. Because the agents' preference satisfies generalized risk sensitivity as defined in Ai and Bansal (2018), the first channel indicates that the resolutions of uncertainty from announcements are associated with realizations of a substantial amount of equity premium. However, Ai and Bansal (2018) can not capture the other two effects since they focus on the economy with a representative agent. Under CRRA utility, I find that the other two channels generate a negative announcement premium no matter with a high IES or a low IES under reasonable belief dynamics. This implies that a more significant generalized risk sensitivity is required to account for the announcement premium in the framework with heterogeneous beliefs.

Furthermore, to measure the effects of the transparency of FOMC announcements, I study a counterfactual economy where the transparency of all future announcements reduces by half. I find the average long-run disagreement increases 0.31%: the optimists' expected growth rate on average increases from 1.95% to 2.11%, while the pessimist' expected growth rate decreases from 1.1% to 0.95%. The implied average stock market volatility is 16.8% higher due to more speculative trading out of more disagreement. The consequences of long-run belief divergence and the associated asset market fluctuations underline the importance of transparency.

Related literature

This paper relates to several aspects of literature. It is related to a long list of papers that study heterogeneous beliefs in general equilibrium, such as Basak (2000, 2005) under CRRA utility with learning and Borovicka (2018) under recursive utility with dogmatic disagreement; however, there are two important differences.[2] First, literature has studied that the trading volume increases in disagreement with more aggressive holdings, whereas the other

[2] A short list is Scheinkman and Xiong (2003), David (2008), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Bhamra and Uppal (2014), Baker, Hollified, and Osambela (2016), Collin-Dufresne, Johannes, and Lochstoer (2017), and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018), Atmaz and Basak (2018).

channel—the trading volume may come from the needs of rebalancing—is largely ignored.[3] In contrast, my paper provides asset-market-based evidence to show that the huge trading volume upon announcements is caused by less disagreement. Also, this is the main economic mechanism in the model to explain the associated trading volume. Second, the disagreement literature focuses on the information coming from macroeconomic quantities, and very little attention has been paid to FOMC announcements. My paper studies the impact of both information sources, which can be easily extended to study other events, such as earnings announcements and other macroeconomic announcements.[4]

My paper is related to the broader literature on the macroeconomic announcement.[5] Savor and Wilson (2013) document a significant equity market return on days with major macroeconomic announcements. Lucca and Moench (2015) find the trading volume of E-mini increases significantly upon FOMC announcements. To explain the underlying mechanism, I document both call and put open interest of SPX decrease significantly at the end of announcement days, which implies the associated trading volume comes from the needs of balancing out of less disagreement. The information channel I emphasize is consistent with recent work by Nakamura and Steinsson (2018). They provide empirical evidence and develop a theoretical model to show that Fed announcements affect beliefs not only about monetary policy but also about other economic fundamentals.

The theoretical model builds on recent developments in asset pricing models for the macroeconomic announcement premium. Ai and Bansal (2018) demonstrate that, the substantial equity market returns realized on FOMC announcement days imply that preferences must satisfy generalized risk sensitivity in a representative agent economy.[6] My paper extends it to the case where agents have heterogeneous beliefs, which not only provides the necessary ingredients to capture the change of beliefs and explain the associated trading vol-

[3]For the related papers, see Buraschi and Jiltsov (2006) and Heyerdahl-Larsen and Illeditsch (2019), among others.

[4]In addition, this paper is connected to literature that demonstrates the importance of heterogeneous beliefs to account for asset market dynamics, such as Bakshi, Madan and Panayotov (2010, 2015) and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018).

[5]Related empirical papers include, among others, Brusa, Savor, and Wilson (2015), Bollerslev, Li, and Xue (2018), Cieslak, Morse, and Vissing-Jorgensen (2019), Boguth, Gregoire, and Martineau (2019), Hu, Pan, Wang, and Zhu (2019), Law, Song, and Yaron (2019).

[6]Ai, Bansal, Im, and Ying (2018) and Wachter and Zhu (2018) develop quantitative models of the announcement premium with a representative agent, based on the generalized risk sensitivity in Ai and Bansal (2018).

ume,[7] but also allows me to study the impact of generalized risk sensitivity under a more realistic framework with heterogeneous agents.[8]

The computation builds on the finite difference method from Achdou et al. (2017) and Ahn et al. (2018). Under aggregate uncertainties, they take a first-order Taylor expansion of the equilibrium conditions around the steady state, which is not plausible to study asset pricing models, especially models with discontinuous value functions upon announcements. In this paper, I solve a 3-dimensional PDE globally without any approximation, which accurately captures the asset market fluctuations. This method can be easily revised to solve other continuous-time models since their Hamilton-Jacobi-Bellman (HJB) equations share a similar mathematical structure.

The rest of the paper is organized as follows. I provide some asset-market-based evidence around FOMC announcements in Section 2. In Section 3, I present a general equilibrium model with heterogeneous beliefs under recursive utility. I explicitly capture endogenous disagreement dynamics. Section 4 analyzes the social planner problem and studies the optimal allocations. In Section 5, I derive the asset pricing implications that arise from time-varying disagreement. In sections 6, I present quantitative implications after calibrating to several low-frequency and high-frequency moments. Section 7 concludes.

2 Empirical Evidence

Given that the FOMC has been improving the way to convey information, I focus on announcements starting from the year 2011. It results in a more precise estimation of the current policies as well as their future impact for two reasons. First, to ”provide additional transparency and accountability” (Bernanke, 2011), the Chair of the Board of Governors

[7]To account for the pre-FOMC drift, Cocoma (2020) studies a model where both the risk and disagreement are very low before announcements and very high after announcements. However, the risk pattern is the opposite of Hu, Pan, Wang, and Zhu (2020), that find the risk (measured by the VIX index) starts to increase six days before announcements, then decreases from 24 hours before FOMC news until the end of days with announcements. Also, this paper shows that both call and put open interest decrease significantly at the end of days with announcements, which can not be explained by higher disagreement after FOMC news.

[8]Many works of literature highlight the importance of heterogeneity, which has different impacts as a representative agent economy. For example, Kaplan, Moll, and Violante (2018), Auclert (2019), and Kekre and Lenel (2020) evaluate the role of redistribution in the transmission mechanism of monetary policy through heterogeneous agent New Keynesian (HANK) models.

holds a press conference following half of the announcements since April 2011. At these meetings, the FOMC also releases the summary of its members' economic projections (SEP), so that three forms of communication take place: the FOMC statement, the SEP, and the press conference with the Chair. Second, as shown later, the absence of pre-FOMC drift and market uncertainty reduction (measured by VIX) before FOMC announcements since 2011 indicates that no information comes out earlier.[9,10] It implies that the market only responds to FOMC upon announcements rather than ahead, which provides a necessary condition to measure the information effect via the asset-market-evidence upon FOMC announcements. I summarize the main findings below and provide details about the data construction in Appendix A.

1. The trading volume of E-mini S&P 500 futures (E-mini) immediately increases a lot upon FOMC announcements.

   In Figure 1, I plot the five-minute trading volume of E-mini from 3 hours before announcements to 1.5 hours after announcements.[11] To eliminate the daily trading pattern, I calculate the relative average trading volume comparing to that at the same time on days without announcements. Before announcements, the trading volume is not significantly different from other days. The trading volume spikes at the announcements, which immediately increases by 3.6 times and keeps at a very high level until 1.5 hours after announcements. This evidence is consistent with the previous literature.[12]

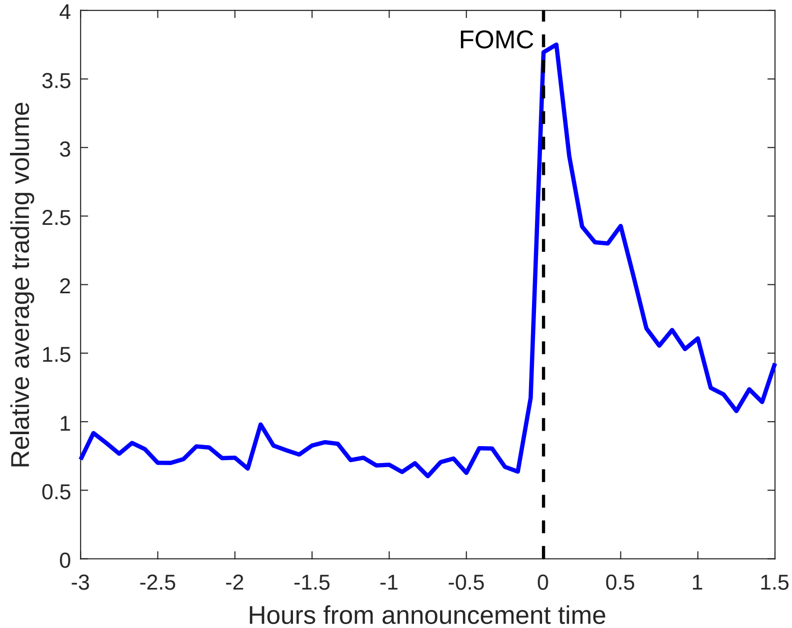
[9] Cieslak, Morse, and Vissing-Jorgensen (2019) and Vissing-Jorgensen (2020) provide a history of leak discussions in FOMC documents and conclude the Fed policymakers would leak information to drive market expectations. Ying (2020) jointly accounts for the pre-FOMC drift and the uncertainty reduction preceding FOMC announcements where the risk is reduced through private information. Hu, Pan, Wang, and Zhu (2020) interpret this as evidence that all investors observe part of the FOMC news.

[10] The Fed pays more attention to prevent leaks from FOMC participants since 2011. For example, the January 2011 FOMC meeting had leaks on the agenda and the transcripts contain a lengthy discussion of the issue (p.5-10 and 197-230): https://www.federalreserve.gov/monetarypolicy/files/FOMC20110126meeting.pdf.

[11] The reason I stop at 1.5 hours is that some FOMC announcements are released at 2:15 PM EST. The E-mini S&P 500 futures have a trading halt from 4:15 PM - 4:30 PM EST. I start from 3 hours before announcements since some FOMC announcements are released at 12:30 PM EST.

[12] For example, Lucca and Moench (2015) document the high trading volume of E-mini upon FOMC announcements between 1997 and 2011.

Figure 1: Trading volume of E-mini around FOMC announcements

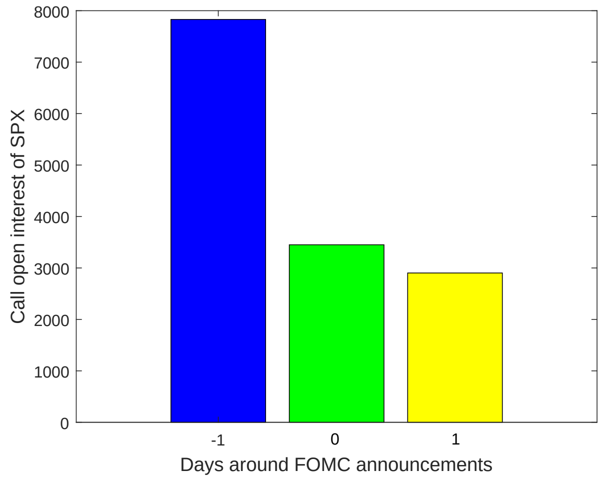


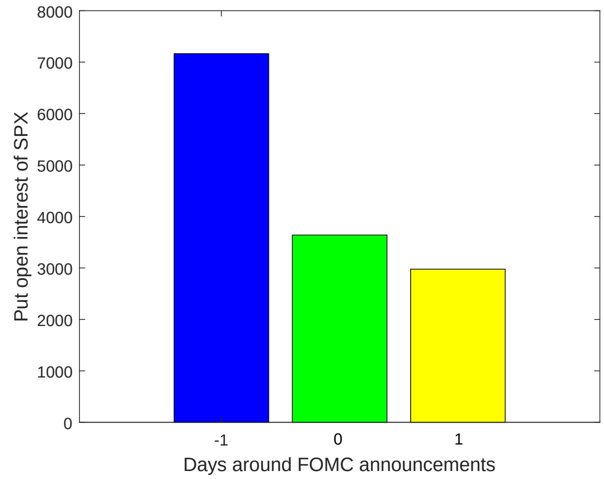
This figure shows the intraday trading volume E-mini S&P 500 futures (E-mini) on FOMC announcement days relative to that at the same time on non-announcement days starting from the year 2011. The trading volume is calculated by the total traded share in 5 minutes. I use as a benchmark all non-announcement days in the prior 21 trading days or since the last announcement, whichever is fewer.

The high trading volume demonstrates that investors' beliefs change a lot when macroeconomic information is released. There are two possible explanations for this empirical evidence. One explanation is that investors interpret the public news differently so that their disagreement increases after announcements, which leads to more aggressive trading in their previous positions. The second explanation is that investors disagree less since they update the beliefs from observing the same public news. The need for rebalancing and unwinding their previous positions contributes to the huge trading volume. The two opposite explanations have different signs of beliefs' dynamics, which would lead to different policy implications of FOMC announcements. Therefore, it is very important to figure out the main channel.

2. Both the call and put open interest of S&P 500 Index Options (SPX) decrease significantly after announcements.

Figure 2: Open interest of SPX before, on, or after FOMC news





This figure shows the level of call and put open interest of S&P 500 Index Options (SPX) prior to, on, and after scheduled FOMC announcements starting from the year 2011. To measure the instantaneous effects of announcements, I focus on the options with the maturity less than 7 days. Table 1 reports the related statistics for the change of open interest.

Figure 2 shows that both call and put open interest of SPX decrease significantly at the end of the day with announcements.[13] To measure the instantaneous effects of announcements, I focus on the options with a maturity 7 days or less.[14] Large amounts of both call and put open interest indicate investors take bets on the incoming FOMC news before announcements and hold the positions according to their beliefs. The announcements carry additional information of the underlying fundamentals, which update their beliefs. The call (put) open interest reduces -53.5% (-47.1%) at the end of the announcement day, which is statistically significant at the 1% level as shown in Table 1. This suggests, they decide to unwind their positions due to less disagreement after announcements.[15] Given the financial markets are complete, this implies the trading volume of stocks upon announcements mainly comes from less disagreement.

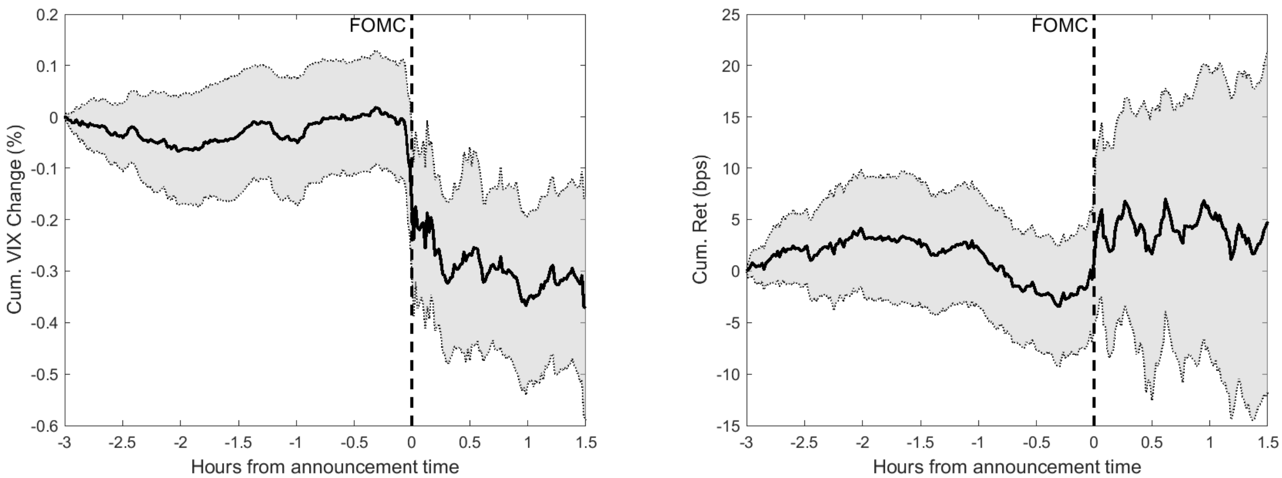
[13]Open interest is only available at daily frequency since it is officially posted by the Options Clearing Corporation (OCC) in the next day's morning.

[14]The results hold robustly for other maturities, such as less than 14 days or 21 days.

[15]Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018) use monthly open interest in interest rate futures to measure the trading on inflation disagreement. They find open interest is increasing in inflation disagreement.

3. Starting from the year 2011, there is no pre-FOMC drift and market uncertainty reduction (measured by VIX) before FOMC announcements. The uncertainty reduces significantly after announcements, which is associated with the realizations of the announcement premium.

Figure 3: Uncertainty reduction and Cum. return around FOMC announcements



The left panel shows average cumulative VIX change around FOMC announcements since 2011. The right panel shows the cumulative return of E-mini around announcements during this period. Shaded areas represent pointwise 95% confidence bands around FOMC means.

The change of disagreement upon announcements leads to asset price dynamics. To better discipline the dynamics of beliefs, I also study the stock return upon announcements at the same period. The right panel of Figure 3 shows that the average equity premium is around 3.8 basis points during the window when investors trade a lot, while the excess return before announcements is not significantly different from zero. This evidence is consistent with Kurov, Wolfe, and Gilbert (2019) that establish the disappearing of pre-FOMC announcement drift starting from 2011. Besides, the left panel of Figure 3 shows the market uncertainty only reduces after FOMC since 2011, which indicates that no information is received by the market before announcements. Therefore, I can precisely estimate the effect of FOMC news on the dynamics of heterogeneous beliefs through the asset market responses upon announcements.

3 Model

I consider a continuous-time, pure-exchange security market economy with a infinite horizon. There is a single consumption good which serves as the numeraire. Agents have identical recursive preferences but differ in their beliefs about the endowment growth rate, which are updated through learning.

3.1 Information structure and heterogeneous beliefs

The stochastic structure of the economy is given by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ with an augmented filtration defined by a family of σ-algebras $\{\mathcal{F}_t\}$, $t \geq 0$ generated by a univariate Brownian motion $W = [B_{Y,t}\, B_{\theta,t}]$. $\left\{\mathcal{F}_t^W\right\}$ denotes the augmented filtration generated by $W(t)$, and $\mathcal{H}$ is a σ-field independent of $\left\{\mathcal{F}^W\right\}$. The field $\mathcal{H}$ whose role is to allow for heterogeneity in agents' priors consists of all possible initial beliefs. The complete information filtration $\{\mathcal{F}_t\}$ is the augmentation of the filtration $\mathcal{H} \times \left\{\mathcal{F}_t^W\right\}$.

The investors in the economy observe the dynamics of aggregate endowment Y_t and the volatility σ_y, but the investors do not observe the growth rate θ_t. The aggregate endowment process Y satisfies

$$\frac{dY_t}{Y_t} = \theta_t dt + \sigma_y dB_{Y,t}, \quad Y_0 > 0, \tag{1}$$

with constant σ_y. The growth rate θ_t under agent i's perspective follows

$$d\theta_t = \rho\left(\bar{\theta} - \theta_t\right) dt + \sigma_\theta dB_{\theta,t},$$

where $\rho > 0$ determines the mean reversion rate of the persistent shock. The standard Brownian motion $B_{\theta,t}$ is independent of $B_{Y,t}$.

In this economy, two types of investors—optimists ($i = 1$) and pessimists ($i = 2$)—differ in the long-run mean of growth rate $\bar{\theta}^i$. On non-announcement days, the investors form optimal estimations of the growth rate by filtering the aggregate endowment Y_t through the information filtration $\mathcal{F}_t^W \subset \mathcal{F}_t$, $t \geq 0$. Particularly, the prior beliefs about θ_t at time $t = 0$ are heterogeneous for each investor and, thus, is $\mathcal{H}$-measurable. The beliefs of the investors about θ_t are updated in a Bayesian fashion, via $m^i(t) = E^i[\theta_t|\mathcal{F}_t]$, where $E^i[\cdot]$ denotes the expectation relative to the subjective probability measure P^i, which is equivalent to the true measure P (which may disagree on $\mathcal{H}$, so that investors have heterogeneous prior beliefs).

Timing of information and Bayesian learning The pre-scheduled announcements come every T days. At time 0, the agent's prior belief about θ_0 can be represented by a normal distribution with mean $m^i(0)$ and variance $Q^i(0)$. On the days without announcements $t \in ((n-1)T, nT)$, investor i updates his beliefs based on the observed endowment process using the Kalman-Bucy filter (Lipster and Shiryaev (2001)):

$$dm^i(t) = \rho\left(\bar{\theta}^i - m^i(t)\right)dt + \frac{Q^i(t)}{\sigma_y}d\tilde{B}^i_{Y,t} \tag{2}$$

$$= \rho\left(\bar{\theta}^i - m^i(t)\right)dt + \frac{Q^i(t)}{\sigma_y^2}\left[\frac{dY_t}{Y_t} - m^i(t)\,dt\right], \quad i = 1, 2. \tag{3}$$

The posterior variance $Q^i(t) = E^i\left[\left(\theta_t - m^i(t)\right)^2|\mathcal{F}_t\right]$ follows

$$dQ^i(t) = \left[\sigma_\theta^2 - 2\rho Q^i(t) - \frac{1}{\sigma^2}Q^i(t)^2\right]dt, \tag{4}$$

which is a deterministic function of time t. Therefore, it's straightforward to show if both agents have the same prior variance at beginning (i.e. $Q^1(0) = Q^2(0)$), they will always have the same posterior variance along the path.

The innovation process of each agent is such that given his perceived growth rate, $m^i(t)$, the observed aggregate endowment obeys

$$dY_t = Y_t\left[m(t)\,dt + \sigma_y d\tilde{B}_{Y,t}\right] = Y_t\left[m^i(t)\,dt + \sigma_y d\tilde{B}^i_{Y,t}\right], \ i = 1, 2, \tag{5}$$

and hence indeed "agrees" with the process he observes. Equation (5) implies that that agent i views the evolution of the Brownian motion as distorted by a drift component $\bar{\mu}^i(t)$, i.e.,

$$d\tilde{B}_{Y,t} = \bar{\mu}^i(t)\,dt + d\tilde{B}^i_{Y,t}, \quad \bar{\mu}^i(t) = \frac{m^i(t) - m(t)}{\sigma_y}, \tag{6}$$

where $\tilde{B}^i_{Y,t}$ is a Brownian motion under P^i by Girsanov's theorem. Consequently, the aggregate endowment is perceived to contain an additional drift component $\bar{\mu}^i(t)\,\sigma_y$, and $\bar{\mu}^i(t)$ can be interpreted as a degree of optimism (if $\bar{\mu}^i(t) > 0$) or pessimism (if $\bar{\mu}^i(t) < 0$) about aggregate endowment relative to the objective perspective. By the Girsanov theorem, the agent i's subjective measure can be generated from the density:

$$\left(\frac{dP^i}{dP}\right)_t = Z^i_t = \exp\left\{-\frac{1}{2}\int_0^t \left(\bar{\mu}^i(s)\right)^2 ds + \int_0^t \bar{\mu}^i(s)\, d\tilde{B}_{Y,s}\right\}, \tag{7}$$

where the martingale Z^i_t measures the disparity between the subjective and objective measure and is commonly called the belief ratio.

On the days with announcements, $t \in \{nT,\ n \geq 1\}$, additional signals about θ_t are revealed through announcements. For $n = 1, 2, 3, \cdots$, we denote s_n as the signal observed at time nT and assume

$$s_n = \theta_{nT} + \zeta_n,$$

where ξ_n is i.i.d. over time, and normally distributed with mean zero and variance σ_S^2. The agents update their beliefs using Bayes' rule:

$$m^{i,+}_{nT} = Q^{i,+}_{nT}\left[\frac{1}{\sigma_S^2}s_n + \frac{1}{Q^{i,-}_{nT}}m^{i,-}_{nT}\right]; \quad \frac{1}{Q^{i,+}_{nT}} = \frac{1}{\sigma_S^2} + \frac{1}{Q^{i,-}_{nT}}, \tag{8}$$

where $m^{i,+}_{nT}$ and $Q^{i,+}_{nT}$ are the posterior mean and variance after announcements, and $m^{i,-}_{nT}$ and $Q^{i,-}_{nT}$ are the posterior mean and variance before announcements, respectively. A special case is that when $\sigma_S^2 = 0$, the announcements can completely reveal the information about θ_t so that $m^{i,+}_{nT} = \theta_{nT}$. This implies that after announcements, there is no disagreement between the two agents under this extreme case.

Proposition 1. *The disagreement among the two agents $m^1(t) - m^2(t)$ is deterministic with explicit solution when the two agents have the same prior variance:*

(i) During the days without announcements, the law of motion of the disagreement is

$$dm_t^1 - dm_t^2 = \left[\underbrace{\rho\left(\bar{\theta}^1 - \bar{\theta}^2\right)}_{\text{the added disagreement}} - \underbrace{\left(\rho + \frac{Q(t)}{\sigma_y^2}\right)\left(m_t^1 - m_t^2\right)}_{\text{the learning component}} \right] dt, \tag{9}$$

which is a first order differential equation. The solution is

$$m_t^1 - m_t^2 = \rho\left(\bar{\theta}^1 - \bar{\theta}^2\right) e^{A(t)} \int_0^t e^{-A(s)} ds + e^{-A(t)} D$$

with $A(t) = \int_0^t \left(\rho + \frac{Q(u)}{\sigma_y^2}\right) du$*, where* $D = m_0^1 - m_0^2$ *is the initial disagreement.*

(ii) When the announcement comes, the disagreement becomes smaller and follows

$$m_{nT}^{1,+} - m_{nT}^{2,+} = \underbrace{\frac{Q_{nT}^+}{Q_{nT}^-}}_{\text{the learning component}} \left(m_{nT}^{1,-} - m_{nT}^{2,-}\right) < m_{nT}^{1,-} - m_{nT}^{2,-}. \tag{10}$$

Proposition 1 shows that on non-announcement days, the dynamics of disagreement have two components: the learning component from the information of aggregate output reduces disagreement, while different long-run means of growth rate increase disagreement. On days with announcements, the FOMC news reduces disagreement since the agents observe the same signal and update their beliefs using Bayes' rule. Besides, from equation (10), the sign of the disagreement never flips after announcements due to the non-negative posterior variance.[16] In other words, if agent 1 is more optimistic than agent 2 before announcements, he is still more optimistic than agent 2 after announcements as long as the FOMC news does not fully reveal the growth rate. When the FOMC news fully reveals the growth rate, the disagreement becomes to be zero. The disagreement keeps increasing until the next announcement and does not vanish in the long run. Furthermore, the initial long-run disagreement can be determined through $m_{nT}^{i,+} = m_0^i$.

[16]This is consistent with Giglio, Maggiori, Stroebel, and Utkus (2019), who find that individuals have large and persistent heterogeneity in beliefs and show strong willingness to agree to disagree.

3.2 The Asset Market

The assets are modeled as in the Lucas (1978) tree economy. I normalize the total supply of risky assets to be one unit. The riskless bond is in zero net supply. I denote the processes $\{P_t\}$ and $\{r_t\}$ as the risky asset price and interest rate processes, respectively.

When $t \in ((n-1)\,T,\, nT)$, the total return on the risky asset is

$$\begin{aligned} dR_t = \frac{Y_t dt + dP_t}{P_t} &= \mu_{R,t} dt + \sigma_{R,t} d\tilde{B}_{Y,t} \\ &= \mu^i_{R,t} dt + \sigma^i_{R,t} d\tilde{B}^i_{Y,t}, \end{aligned} \tag{11}$$

where $\mu^i_{R,t}$ and $\sigma^i_{R,t}$ are endogenously determined in the equilibrium, which represents the risky security dynamics as perceived by investor i.

Proposition 2. *The agreement of price level across investors implies that on days without announcements:*

(i) Agents have the same perceived diffusion of the return dynamics:

$$\sigma_{R,t} \equiv \sigma^i_{R,t}, \quad t \in ((n-1)\,T,\, nT)$$

(ii) The perceived drift of the return dynamics follows

$$\mu^i_{R,t} = \mu_{R,t} + \bar{\mu}^i\,(t)\,\sigma_{R,t}, \quad t \in ((n-1)\,T,\, nT)$$

Proposition 2 shows the difference of the perceived return is determined by differences of opinion about the expected growth rate as shown in equation (6). Optimists' perceived return is higher under their subjective measure since they expect more cash flows in the future.

At announcements $t = nT$, the instantaneous return is determined by the price change

$$\frac{R^+_t}{R^-_t} = \frac{P^+_t}{P^-_t},$$

where the dividend part goes to zero when $dt \to 0$.

3.3 Agents' preference and optimization

I assume that both agents are endowed with a Kreps-Porteus preference with risk aversion γ and intertemporal elasticity of substitution ψ. In continuous time, the preference is represented by a stochastic differential utility, which can be specified by a pair of aggregators $(f, \mathcal{A})$ such that in the interior of $(nT, (n+1)\,T)$,

$$dV_t = [-f(C_t, V_t) - \frac{1}{2}\mathcal{A}(V_t)||\sigma_V(t)||^2]dt + \sigma_V(t)dB_t \tag{12}$$

I adopt the convenient normalization $\mathcal{A}(v) = 0$, and denote $\bar{f}$ the normalized aggregator. Under this normalization, $\bar{f}(C, V)$ is:

$$\bar{f}(C, V) = \frac{\beta}{1 - 1/\psi} \frac{C^{1-1/\psi} - ((1-\gamma)\,V)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)\,V)^{\frac{1-1/\psi}{1-\gamma}-1}}. \tag{13}$$

The case of $\psi = 1$ is obtained as the limit of (13) with $\psi \to 1$:

$$\bar{f}(C, V) = \beta V\,[(1-\gamma)\ln C - \ln[(1-\gamma)\,V]]\,.$$

Because announcements typically result in discrete jumps in the posterior belief about θ_t, the value function is typically not continuous at announcements. For $t = nT$, the pre-announcement utility and post-announcement utility are related by

$$V_t^{i,-} = E_t^{i,-}\left[V_t^{i,+}\right] \tag{14}$$

where $E_t^{i,-}$ represents agent i's expectation with respect to the pre-announcement information at time t.

Both agents choose his consumption rate and the portfolio decision under his subjective measure to solve

$$
\begin{aligned}
V^i &= \max_{\{C_t^i, \pi_t^i\}} E^i \left[\int_0^\infty \bar{f}(C_s^i, V_s^i) dt \right] \\
\text{s.t. } \frac{dW_t^i}{W_t^i} &= \left[r_t + \pi_t^i \left(\mu_{R,t}^i - r_t \right) - \frac{C_t^i}{W_t^i} \right] dt + \pi_t^i \sigma_{R,t}^i d\tilde{B}_{Y,t}^i,
\end{aligned} \tag{15}
$$

where π_t^i is the ratio of the risky asset holdings of agent i to its total wealth W_t^i.

3.4 The equilibrium

Definition. An equilibrium is a set of price processes $\{P_t\}$ and $\{r_t\}$, and decisions $\{C_t^1, C_t^2, \pi_t^1, \pi_t^2\}$ such that

1. Given the price processes, decisions solve the consumption-savings problems of both agents (15) under their subjective measures, associated with the boundary condition (14) upon announcements.

2. The risky asset market clears

$$
\pi_t^1 W_t^1 + \pi_t^2 W_t^2 = P_t \tag{16}
$$

3. The goods market clears:

$$
C_t^1 + C_t^2 = Y_t \tag{17}
$$

Given market clearing in risky asset and goods markets, the bond market clears by Walras' law. Finally, an equilibrium relation that proves useful when deriving the solution is that

$$
W_t^1 + W_t^2 = P_t. \tag{18}
$$

That is, since bonds are in zero net supply, the wealth of both agents must sum to the value of the risky asset.

4 Planner's problem and optimal allocations

Since the welfare theorems hold in the economy, in this section I characterize optimal allocations from the social planner's problem.

4.1 The planner's problem

I utilize a characterization based on the more general variational utility approach studied by Geoffard (1996) and Dumas, Uppal, and Wang (2000). They show that recursive preferences for each agent i can be represented as a solution to the maximization problem

$$\lambda_t^i V_t^i \left(C^i\right) = \sup_{\left\{\nu_s^i\right\}_{s\geq t}} E_t^i \left[\int_t^\infty \lambda_s^i F\left(C_s^i, \nu_s^i\right) ds\right] \tag{19}$$

subject to

$$d\lambda_t^i = -\nu_t^i \lambda_t^i dt,\ \lambda_0^i > 0 \tag{20}$$

where ν^i is the discount rate process, and λ^i is the discount factor process. For the case of the Duffie–Epstein–Zin preferences, the felicity function $F\left(C, \nu\right)$ is given by

$$F\left(C, \nu\right) = \beta \frac{C^{1-\gamma}}{1-\gamma} \left(\frac{1 - \frac{1-\frac{1}{\psi}}{1-\gamma}\frac{\nu}{\beta}}{1 - \frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{1-\frac{1-\gamma}{1-\frac{1}{\psi}}}$$

I follow Dumas et al. (2000) and Borovicka (2018), and introduce a fictitious planner who maximizes a weighted average of the continuation values of the two agents. The planner's value function is the solution to the problem

$$\begin{aligned} J_0\left(\lambda_0^1, \lambda_0^2, Y_0, m_0\right) &= \sup_{\left(C^1, C^2\right)} \lambda_0^1 V_0^1\left(C^1\right) + \lambda_0^2 V_0^2\left(C^2\right) \qquad (21) \\ &= \sup_{\left(C^1, C^2, \nu^1, \nu^2\right)} \left\{E_0\left[\int_0^\infty \lambda_s^1 F\left(C_s^1, \nu_s^1\right) ds\right] + E_0\left[\int_0^\infty \lambda_s^2 F\left(C_s^2, \nu_s^2\right) ds\right]\right\} \end{aligned}$$

subject to:

$$
\begin{aligned}
d\lambda_t^1 &= \lambda_t^1 \left[-\nu_t^1 dt + \bar{\mu}^1(t)\, d\tilde{B}_{Y,t} \right],\ \lambda_0^1 > 0, \\
d\lambda_t^2 &= \lambda_t^2 \left[-\nu_t^2 dt + \bar{\mu}^2(t)\, d\tilde{B}_{Y,t} \right],\ \lambda_0^2 > 0, \\
\frac{dY_t}{Y_t} &= m(t)\, dt + \sigma_y d\tilde{B}_{Y,t}, \\
dm(t) &= \rho \left(\bar{\theta} - m(t) \right) dt + \frac{Q(t)}{\sigma_y} d\tilde{B}_{Y,t}, \\
C_t^1 + C_t^2 &= Y_t,
\end{aligned}
$$

where I write the planner's objective function under objective measure, without loss of generality. For $t = nT$, the pre-announcement utility and post-announcement utility are related by

$$
J_t^- = E_t^- \left[J_t^+ \right].
$$

4.2 The HJB equation

Due to the homogeneity of the value function,

$$
J(\lambda_1, \lambda_2, Y, m, t) = Y^{1-\gamma} (\lambda_1 + \lambda_2)\, \tilde{J} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2}, m, t \right) = Y^{1-\gamma} \theta_2 \tilde{J} \left(\varpi^1, m, t \right)
$$

where $\varpi^1 = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $\varpi^2 = \lambda_1 + \lambda_2$. ϖ^1 represents the Pareto share of agent 1, which is obviously bounded between zero and one.

Proposition 3. *The planner's problem can be characterized as a solution to*

$$
\begin{aligned}
0 = {} & \varpi^1 \frac{\beta}{1-\frac{1}{\psi}} \left(\zeta^1\right)^{1-\frac{1}{\psi}} \left((1-\gamma)\tilde{J}^1\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \left(1-\varpi^1\right) \frac{\beta}{1-\frac{1}{\psi}} \left(1-\zeta^1\right)^{1-\frac{1}{\psi}} \left((1-\gamma)\tilde{J}^2\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} \\
& + \tilde{J}_t + \left(-\frac{\beta(1-\gamma)}{1-\frac{1}{\psi}} + m(1-\gamma) + \left(\varpi^1 \bar{\mu}^1(t) + \left(1-\varpi^1\right)\bar{\mu}^2(t)\right)(1-\gamma)\sigma_y - \frac{1}{2}\gamma(1-\gamma)\sigma_y^2 \right) \tilde{J} \\
& + \varpi^1 \left(1-\varpi^1\right) \left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)(1-\gamma)\sigma_y \tilde{J}_{\varpi^1} + \frac{1}{2}\left(1-\varpi^1\right)^2 \left(\varpi^1\right)^2 \left(\bar{\mu}^1(t)-\bar{\mu}^2(t)\right)^2 \tilde{J}_{\varpi^1\varpi^1} \\
& + \left[\rho\left(\bar{\theta}-m\right) + (1-\gamma)Q(t) + \left(\varpi^1\bar{\mu}^1(t) + \left(1-\varpi^1\right)\bar{\mu}^2(t)\right)\frac{Q(t)}{\sigma_y}\right] \tilde{J}_m + \\
& \varpi^1\left(1-\varpi^1\right)\left(\bar{\mu}^1(t)-\bar{\mu}^2(t)\right)\frac{Q(t)}{\sigma_y}\tilde{J}_{\varpi^1 m} + \frac{1}{2}\left(\frac{Q(t)}{\sigma_y}\right)^2 \tilde{J}_{mm},
\end{aligned}
\tag{22}
$$

with the following boundary conditions

(i) $\tilde{J}(0,m,t) = H^2(m,t)$ *and* $\tilde{J}(1,m,t) = H^1(m,t)$*, where* $H^i(m,t)$ *are the solution to the following equation*

$$
\begin{aligned}
0 = {} & \frac{\beta}{1-\frac{1}{\psi}} \left[(1-\gamma)H^i\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + H_t^i + \left[-\frac{\beta(1-\gamma)}{1-\frac{1}{\psi}} + (1-\gamma)\left(m + \sigma_y \bar{\Pi}^i(t)\right) - \frac{1}{2}\gamma(1-\gamma)\sigma_y^2\right] H^i \\
& + \left[\rho\left(\bar{\theta}-m\right) + (1-\gamma)Q(t) + \bar{\Pi}^i(t)\frac{Q(t)}{\sigma_y}\right] H_m^i + \frac{1}{2}\frac{Q^2(t)}{\sigma_y^2} H_{mm}^i
\end{aligned}
\tag{23}
$$

where $\bar{\Pi}^i(t) = \bar{\mu}^i(t)$*. This corresponds to the economy where only one agent exists in the long run.*

(ii) The boundary of m *is reflected since* m_t *follows the standard Ornstein–Uhlenbeck process.*

(iii) For all $n = 1, 2, \ldots$

$$
\tilde{J}\left(\varpi^1, m^-, nT^-\right) = E_{nT}^-\left[\tilde{J}\left(\varpi^1, m^+, nT^+\right)\right].
$$

In equation (22), the functions $\tilde{J}^n\left(\varpi^1, m, t\right)$ are the continuation values of two agents scaled by $Y^{1-\gamma}$,

$$\tilde{J}^1\left(\varpi^1, m, t\right) \doteq \tilde{J}\left(\varpi^1, m, t\right) + \left(1-\varpi^1\right)\tilde{J}_{\varpi^1}\left(\varpi^1, m, t\right)$$
$$\tilde{J}^2\left(\varpi^1, m, t\right) \doteq \tilde{J}\left(\varpi^1, m, t\right) - \varpi^1\tilde{J}_{\varpi^1}\left(\varpi^1, m, t\right)$$

The consumption share ζ^1 is given

$$\zeta^1\left(\varpi^1, m, t\right) = \frac{\left(\varpi^1\right)^{\psi}\left[(1-\gamma)\tilde{J}^1\left(\varpi^1, m, t\right)\right]^{\frac{1-\psi\gamma}{1-\gamma}}}{\left(\varpi^1\right)^{\psi}\left[(1-\gamma)\tilde{J}^1\left(\varpi^1, m, t\right)\right]^{\frac{1-\psi\gamma}{1-\gamma}} + \left(1-\varpi^1\right)^{\psi}\left[(1-\gamma)\tilde{J}^2\left(\varpi^1, m, t\right)\right]^{\frac{1-\psi\gamma}{1-\gamma}}},$$

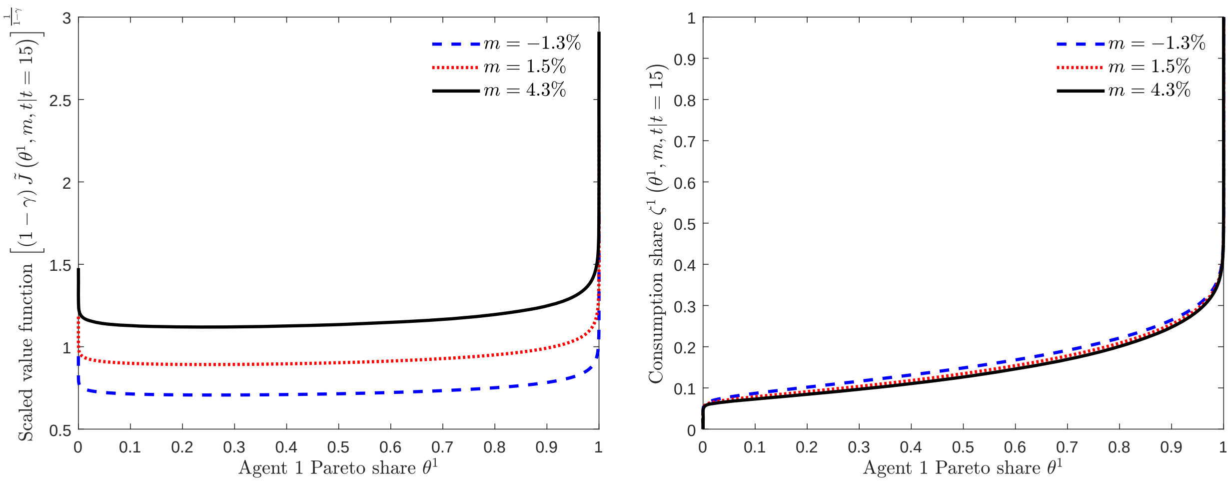
which implies the agent 2's consumption share is $\zeta^2\left(\varpi^1, m, t\right) = 1 - \zeta^1\left(\varpi^1, m, t\right)$. Furthermore, the law of motion of ϖ^1 follows

$$\begin{aligned} d\varpi^1 &= \varpi^1\left(1-\varpi^1\right)\left[\lambda_t^2 - \lambda_t^1 + \left(\varpi^1\bar{\mu}^1(t) + \left(1-\varpi^1\right)\bar{\mu}^2(t)\right)\left(\bar{\mu}^2(t) - \bar{\mu}^1(t)\right)\right]dt \\ &\quad + \varpi^1\left(1-\varpi^1\right)\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)d\tilde{B}_{Y,t} \\ &\doteq \mu_{\varpi^1}\left(\varpi^1, m, t\right)dt + \sigma_{\varpi^1}\left(\varpi^1, m, t\right)d\tilde{B}_{Y,t}, \end{aligned} \tag{24}$$

which is the key variable to determine the long run wealth distribution of the agents, as discussed in Appendix B.

Figure 4 shows the scaled value function $\left[(1-\gamma)\tilde{J}\left(\varpi^1, m, t|t=15\right)\right]^{\frac{1}{1-\gamma}}$ as well as agent 1's consumption share $\zeta^1\left(\varpi^1, m, t|t=15\right)$ as a function of the expected growth rate under objective measure. The left panel shows the limit of the value function converges to the economy where there is only one agent. In other words, when the Pareto share $\varpi^1 = 1$ ($\varpi^1 = 0$), the economy with heterogeneous agents converges to the economy where there is only agent 1 (agent 2). Since agent 1 expects a higher growth rate, the economy where only agent 1 exists has a higher scaled value than that of agent 2. Given the same Pareto share, the scaled value function is increasing in the expected growth rate. The right panel shows agent1's consumption share is a increasing function in the Pareto share, which starts from 0 and converges to 1 when agent 1 dominates in the economy. Given the same Pareto share, the consumption share decreases in the expected growth rate. This comes from the domination of substation effect under a high IES in the baseline calibration. Since agent 1 is more optimistic comparing to agent 2, the substation effect is stronger, which leads to a smaller consumption share when the growth rate increases.

Figure 4: Scaled value function and consumption share



This figure shows that the normalized value function and consumption share as a function of agent 1's Pareto share for the baseline calibration when $t = 15$. The parameter values are reported in Table 2.

5 Disagreement and asset pricing

In this section, I highlight how the dynamics of disagreement affect the trading volume and asset return, especially upon announcements.

5.1 The trading volume of the stock

For $t \in (nT, (n+1)T)$, the stock price P_t is determined by the present value of future dividends

$$P_t = E_t^i\left[\int_t^{(n+1)T} \frac{\Lambda_u^i}{\Lambda_t^i} Y_u du + \frac{\Lambda_{(n+1)T}^i}{\Lambda_t^i} P_{(n+1)T}^-\right] = W_t^1 + W_t^2, \quad \forall i = 1, 2, \tag{25}$$

where the second equation is implied by the market clearing condition (18). Since utility is homogeneous of degree $1-\gamma$, for each agent i, aggregate wealth in units of consumption at time t must be

$$W_t^i = \frac{(1-\gamma)\, V_t^i}{D_C f\left(C_t^i, V_t^i\right)} = \frac{1}{\beta} Y_t \left[\zeta^i\left(\varpi^1, m, t\right)\right]^{\frac{1}{\psi}} \left[(1-\gamma)\, \tilde{J}^i\left(\varpi^1, m, t\right)\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \tag{26}$$

The absolute change of agent 1's position in risky asset (scaled by the asset's value)

$$n_{1,t} = \frac{\pi_t^1 W_t^1}{P_t}, \tag{27}$$

is the trading volume of the stock, which is defined by through the following equation:

$$\mathrm{TV}_t = \left| \int n_{1,t} d\Phi_t - \int n_{1,t-dt} d\Phi_{t-dt} \right| = \left| \int \frac{\pi_t^1 W_t^1}{P_t} d\Phi_t - \int \frac{\pi_{t-1}^1 W_{t-dt}^1}{P_{t-dt}} d\Phi_{t-dt} \right|$$

where Φ_t captures the distribution of agent 1 at time t. Given agent 2's stock holdings is $1 - n_{1,t}$, the measure of trading volume does not depend on which agent I follow.

To study the trading volume, it's important to first understand how the disagreement affects the share holdings. From equation (18) and (27), I rewrite the sharing holdings of stock by agent 1 as

$$n_1 = \frac{\pi^1 W^1}{P} = \underbrace{\pi^1}_{\text{the portfolio share channel}} \underbrace{\frac{W^1}{W^1 + W^2}}_{\text{the wealth accumulation channel}}. \tag{28}$$

This implies that agent 1's share holdings are determined by two elements: (i) the agent 1's portfolio share invested in the risky asset π^1; (ii) the agent 1's wealth share $\frac{W^1}{W^1+W^2}$ in the economy. When agent 1 becomes less optimistic (relative to agent 2), he will reduce his portfolio share of the stock π^1 since the expected return becomes lower. The effect on the wealth share accumulation is ambiguous, which depends on the relative value of risk aversion and IES as pointed in Borovicka (2019). For the trading volume upon announcements, the instantaneous wealth change would be quantitatively negligible comparing to the portfolio share channel.[17] This implies when agent 1 becomes less optimistic (relative to agent 2) after announcements, he would like to reduce his holdings of the stock, which leads to the high

[17]See more discussions in section 6.

trading volume in the stock market.

5.2 The risk premium

Using the construction from Duffie and Epstein (1992), the SDF process for agent 1 under his subjective belief is given by

$$\Lambda^1 = \exp\left(-\int_0^t \nu_s^1 ds\right) D_C F\left(C_t^1, \nu_s^1\right)$$
$$= \beta \exp\left(-\int_0^t \nu^1\left(\varpi^1, m, s\right) ds\right) Y_t^{-\gamma}\left[\zeta^1\left(\varpi^1, m, t\right)\right]^{-\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^1\left(\varpi^1, m, t\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} \quad (29)$$

The state price density at time t depends on the Pareto weight θ_t^1, the posterior mean m, the deterministic posterior variance $Q(t)$ and disagreement $\bar{\mu}^i(t)$, as well as the fundamental risk in Y_t. As equation (29) shows, the disagreement affects agent 1's the SDF through three parts: (1) the direct belief distortion from the discount rate process $\nu^1\left(\varpi^1, m, t\right)$; (2) the indirect reallocation of consumption share between the two agents $\zeta^1\left(\varpi^1, m, t\right)$; (3) the indirect information channel through the continuation utility $(1-\gamma)\tilde{J}^1\left(\varpi^1, m, t\right)$.

When $t \in ((n-1)T, nT)$, the pricing kernel Λ^1 is a continuous diffusion process with the law of motion

$$\frac{d\Lambda^1}{\Lambda^1} = -r\left(\varpi^1, m, t\right) dt - \sigma_\Lambda^1\left(\varpi^1, m, t\right) d\tilde{B}_{Y,t}^1.$$

The interest rate is

$$r\left(\varpi^1, m, t\right) = -\Lambda_t^1 + \sigma_\Lambda^1\left(\varpi^1, m, t\right)\bar{\mu}^1(t) + \gamma m - \frac{1}{2}\gamma(\gamma+1)\sigma_y^2$$
$$-\frac{\Lambda_{\varpi^1}^1}{\Lambda^1}\left(\mu_\theta - \gamma\sigma_\theta\sigma_y\right) - \frac{\Lambda_m^1}{\Lambda^1}\left(\mu_m - \gamma\sigma_m\sigma_y\right) - \frac{1}{2}\frac{\Lambda_{\varpi^1\varpi^1}^1}{\Lambda^1}\sigma_\theta^2 - \frac{1}{2}\frac{\Lambda_{mm}^1}{\Lambda^2}\sigma_m^2 - \frac{\Lambda_{\varpi^1 m}^1}{\Lambda^1}\sigma_m\sigma_\theta. \quad (30)$$

In the first line of equation (30), the first term contains the agent 1's endogenous discount rate process $\nu^1\left(\varpi^1, m, s\right)$, which is affected by the dynamics of disagreement and Pareto share. The second term is agent 1's belief deviation relative to objective measure $\bar{\mu}^i(t)$ times his perceived market price of risk. The third term is the wealth effect associated with expected growth rate of aggregate endowment in the absence of aggregate shocks, and the fourth term is the precautionary savings effect associated with aggregate shocks to output. The second line of Equation (30) incorporates the direct impact of disagreement on the state price density. In particular, the first two terms capture an additional wealth effect associated with the expected change of Pareto share ϖ^1 and beliefs updating of output growth m in the absence of aggregate shocks. The last three terms is the precautionary savings effect associated with aggregate shocks in the Pareto share, learning of output growth and their correlation.

The market prices of risk as perceived by the agent 1 and 2 are, respectively:

$$\sigma_\Lambda^1\left(\varpi^1, m, t\right) = \gamma\sigma_y - \frac{1}{\psi}\left[\frac{\zeta^1_{\varpi^1}}{\zeta^1}\sigma_{\varpi^1} + \frac{\zeta^1_m}{\zeta^1}\sigma_m\right] - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left[\frac{\tilde{J}^1_{\varpi^1}}{\tilde{J}^1}\sigma_{\varpi^1} + \frac{\tilde{J}^1_m}{\tilde{J}^1}\sigma_m\right] \tag{31}$$

$$\sigma_\Lambda^2\left(\varpi^1, m, t\right) = \sigma_\Lambda^1\left(\varpi^1, m, t\right) + \bar{\mu}^2\left(t\right) - \bar{\mu}^1\left(t\right). \tag{32}$$

The first term in Equation (31) is the market price of aggregate endowment risk. The second term is the standard market price of speculative risk, and the third term captures the impact of recursive preferences on the market price of risk. The state price density that perceived by the two agents is separated by their disagreement of expected growth rate normalized by the standard deviation of aggregate output, which is a deterministic function of time t. The optimist perceives a higher market price of risk than the pessimist, as equation (32) shows.

The risk premium of the risky asset perceived by agent $i \in \{1, 2\}$ is determined by the exposure to the risk of the stock $\sigma_R\left(\varpi^1, m, t\right)$, and the equilibrium price of this risk given by $\sigma_\Lambda^i\left(\varpi^1, m, t\right)$:

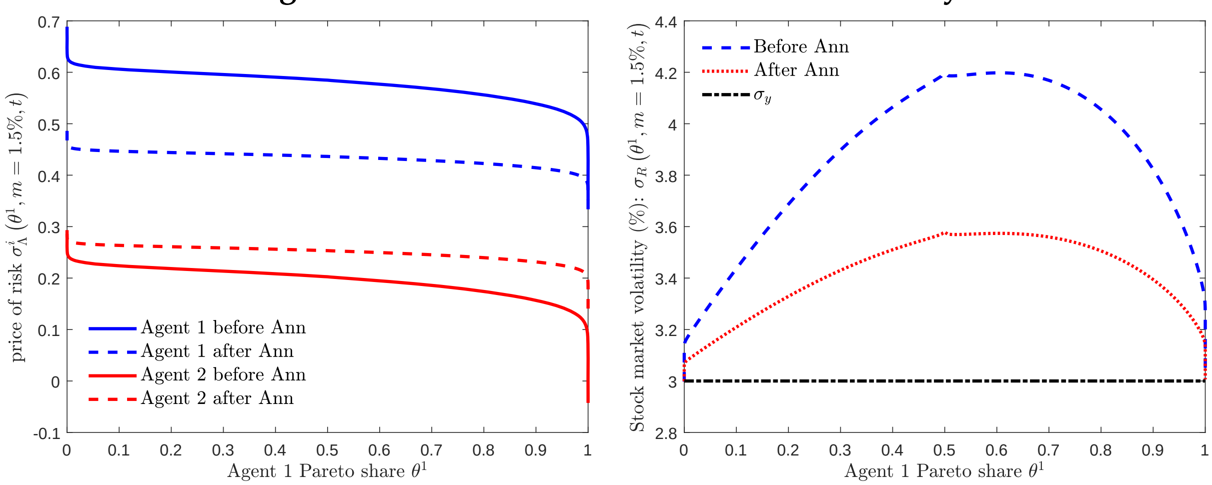
$$\mu_R^i\left(\varpi^1, m, t\right) - r\left(\varpi^1, m, t\right) = \sigma_R\left(\varpi^1, m, t\right)\sigma_\Lambda^i\left(\varpi^1, m, t\right), \tag{33}$$

which implies the difference of the risk premium perceived by the two agents is determined by

$$\mu_R^1\left(\varpi^1, m, t\right) - \mu_R^2\left(\varpi^1, m, t\right) = \sigma_R\left(\varpi^1, m, t\right)\left[\sigma_\Lambda^1\left(\varpi^1, m, t\right) - \sigma_\Lambda^2\left(\varpi^1, m, t\right)\right]$$
$$= \sigma_R\left(\varpi^1, m, t\right)\left(\bar{\mu}^1\left(t\right) - \bar{\mu}^2\left(t\right)\right), \tag{34}$$

where $\sigma_R\left(\varpi^1, m, t\right) = \frac{\left[\zeta^1\right]^{\frac{1}{\psi}}\left[(1-\gamma)\bar{J}^1\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\sigma_{W1}+\left[\zeta^2\right]^{\frac{1}{\psi}}\left[(1-\gamma)\bar{J}^2\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\sigma_{W2}}{\left[\zeta^1\right]^{\frac{1}{\psi}}\left[(1-\gamma)\bar{J}^1\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}+\left[\zeta^2\right]^{\frac{1}{\psi}}\left[(1-\gamma)\bar{J}^2\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}$ as shown in equation (53), which is wealth-weighted wealth volatility of the two agents. The equity premium perceived by optimists is higher, as shown in equations (32) and (34).

Figure 5: Price of risk and stock market volatility



The figure plots the market prices of risk as perceived by both agents and the stock market volatility around announcements. The parameters values are reported in Table 2, where I assume announcement happens monthly.

The left panel of Figure 5 illustrates the market prices of risk as perceived by the optimists (agent 1) and pessimists (agent 2), which are defined in equations (31) and (32). Optimists always have a higher price of risk, where the difference is captured by the relative disagreement at time t: $\bar{\mu}^1\left(t\right) - \bar{\mu}^2\left(t\right)$. After announcements, the difference between the perceived market prices of risk is smaller due to the reduction of disagreement. In particular, agent 2's perceived market price of risk increases after announcements since he becomes less pessimistic. The right panel shows the disagreement leads to excess volatility through speculative trade and the impact of recursive utility so that the stock market is more volatile than the output growth volatility, which corresponds to the second term and the third term in equation (31). The stock market volatility converges to the output growth volatility when the Pareto share approaches either 0 or 1 because in this case investors have nobody to speculate with. Besides, agents have the highest disagreement just before announcements, which amplifies the speculative effect and contributes to a more volatile stock return.

Given the equity premium is calculated under the objective measure (or under the belief of the econometrician) in the data, I present the equity premium under objective measure in the model. On the days without announcements, the price is determined by

$$P_t = E_t^i \left[\frac{\Lambda_{t+dt}^i}{\Lambda_t^i} P_{t+dt} + \int_t^{t+dt} \frac{\Lambda_s^i}{\Lambda_t^i} Y_s ds \right] = E_t \left[\frac{Z_{t+dt}^i}{Z_t^i} \frac{\Lambda_{t+dt}^i}{\Lambda_t^i} P_{t+dt} + \int_t^{t+dt} \frac{Z_s^i}{Z_t^i} \frac{\Lambda_s^i}{\Lambda_t^i} Y_s ds \right],$$

where Z_t^i is the belief ratio defined in equation (7). Since prices are observed in equilibrium, agents have to agree on them. Under the complete market, the objects

$$\frac{Z_{t+dt}^i}{Z_t^i} \frac{\Lambda_{t+dt}^i}{\Lambda_t^i}$$

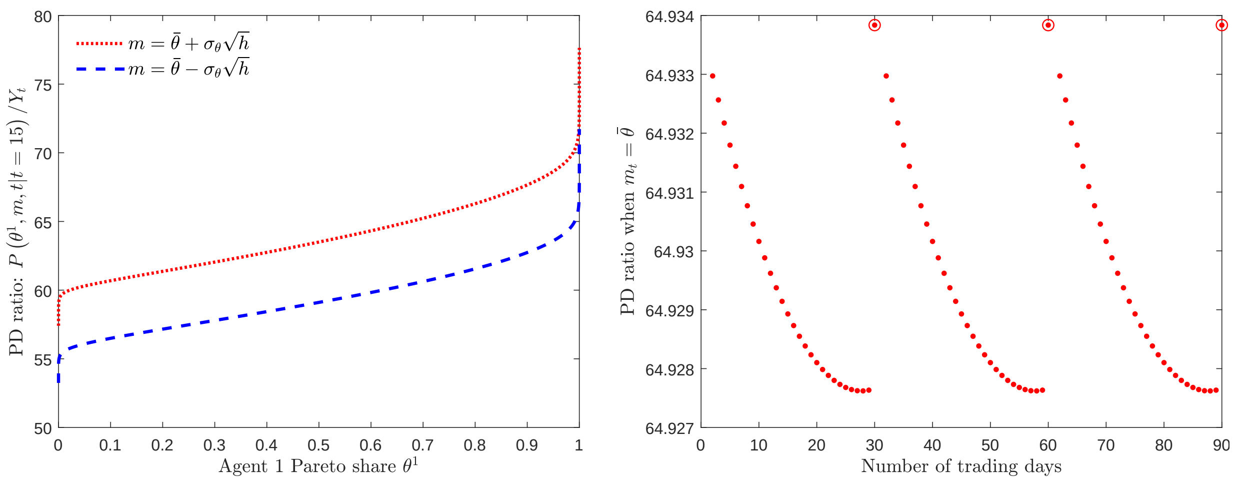
have to be equalized across agents and deviations in beliefs have to be offset by the reciprocal deviations in marginal utilities.

At announcements, the price is determined by

$$P_{nT}^- = E_{nT}^{i,-} \left[\frac{\Lambda_{nT}^{i,+}}{\Lambda_{nT}^{i,-}} P_{nT}^+ \right] = E_{nT}^- \left[\frac{Z_{nT}^{i,+}}{Z_{nT}^{i,-}} \frac{\Lambda_{nT}^{i,+}}{\Lambda_{nT}^{i,-}} P_{nT}^+ \right].$$

Figure 6 plots the price-to-dividend ratio $\frac{P_t}{Y_t}$ in the benchmark calibration. A persistent increase in expected consumption growth provides news about future cash flows and discount rates that go in opposite directions. With a high IES in the benchmark, the cash flow effect dominates, which pushes up the price-to-dividend ratio and the expected return goes down. The price-to-dividend ratio is increasing in agent 1's Pareto share since agent 1 is more optimistic, which amplifies the cash flow effect. The right panel shows the in general, the announcements are associated with an immediate increase in the valuation ratio for a fixed expected growth rate. The price-to-dividend ratio drops gradually until the next announcement. The intuition is that when the announcement is approaching, the uncertainty increases and the disagreement decreases, which leads investors to save more in risk-free assets, lowering the risk-free rate. In the meantime, investors are reluctant to hold risky assets, which increases the equity premium. When the IES is high, the interest rate does not change much, and the risk premium effect dominates. Therefore, under a fixed Pareto share and a fixed expected growth rate, the reduction of uncertainty upon announcements in general pushes up the price-to-dividend ratio in the benchmark calibration, which is associated with realizations of equity premium.

Figure 6: Price-to-dividend ratio



The figure plots the price-to-dividend ratio in the benchmark calibration. The parameter values are reported in Table 2, where I assume the announcement happens monthly. The right panel plots the evolution of price-to-dividend ratio over the announcement cycles under a fixed expected growth rate $m = \bar{\theta}$ and a fixed Pareto share, which is the average Pareto share in the simulations as discussed in section 6. The dotted line is the price-to-dividend ratio on non-announcement days, and the circles indicate the price-to-dividend ratio on announcement days.

The announcement premium under objective measure is determined by the negative covariance of SDF (under objective measure) and price-to-dividend ratio:

$$-Cov_{nT}^{-}\left(\underbrace{\frac{Z_{nT}^{i,+}}{Z_{nT}^{i,-}}}_{\text{Belief dynamics}}\;\underbrace{\frac{\Lambda_{nT}^{i,+}}{\Lambda_{nT}^{i,-}}}_{\text{SDF dynamics}},\frac{P_{nT}^{+}}{P_{nT}^{-}}\right) \tag{35}$$

where $\frac{Z_{nT}^{i,+}}{Z_{nT}^{i,-}}$ measures the the belief ratio dynamics relative to the objective measure. Three channels contribute to the dynamics of SDF upon announcements. The first channel is the informational effect through the continuation utility in the SDF. The second channel comes from the reallocation of consumption between the two agents upon announcements as well as the discount rate process $\nu^1\left(\varpi^1, m, t\right)$, which appear in the SDF dynamics. The third channel is that the announcements reduce the disagreement upon announcements, which affects the SDF under objective measure through the belief dynamics. The belief dynamics directly affect the allocation of asset holdings of the stock market, as discussed in section 6. While the first channel is the same as Ai and Bansal (2018) and Ai, Bansal, Im, and Ying (2019), the other two channels are largely ignored, which only exist in the economy with heterogeneous beliefs.

The first channel positively contributes to the announcement premium positively when $\gamma > \frac{1}{\psi}$. The intuition is the following: since agents have the preference with early resolution of uncertainty, FOMC announcements result in non-trivial reductions of uncertainty, and are associated with realizations of a substantial amount of equity premium, which is consistent with Ai and Bansal (2018). The effects of the other two channels is ambiguous, which can be affected by many elements. For example, the level of IES and the change of belief deviations jointly determine whether the income effect or the substation effect dominates upon announcements. This directly affects the SDF through the reallocation of consumption among the investors. To quantify the effects of the other two channels, I study the announcement premium under CRRA utility in section 6.3.

6 Quantitative results

In this section, I assess the model's ability to replicate the key moments of quantities and prices in both low frequency (annually) and high frequency (upon announcements). For the low frequency, I study the macro quantities, asset price dynamics, and the disagreement among the forecasts of the GDP's annual growth rate. I target the level of announcement premium and the time-series pattern of uncertainty reduction after announcements for the high frequency. Then I study the implications of trading volume and the dynamics of beliefs upon FOMC announcements.

Borovicka (2019) derives conditions for the existence of nondegenerate long-run equilibria in which agents who differ in accuracy of their beliefs coexist in the long run, and show that these equilibria exist for broad ranges of plausible parameterizations when risk aversion is larger than the inverse of the IES. This result also applies to my benchmark calibration. However, in section 6.3, to highlight the importance of recursive preferences, I study economies with CRRA utility, where the nondegenerate equilibria do not exist. For fair comparisons, the model statistics are computed from 1000 parallel samples, and each sample contains 20×360 days of simulated data using the policy functions obtained from the model solutions. The starting Pareto share of the benchmark is 0.5, which indicates the two types of agents equally exist at the beginning.[18] The prior beliefs are set to be the steady states at $t = 0$, which are determined in Proposition 1.

6.1 Calibration

Annual macro quantities, asset prices dynamics, and disagreement Ai and Bansal (2018) identify parameter values to match aggregate macro quantities and asset prices in a model identical to mine, but with only one agent. I adopt the same parameter values as Ai and Bansal (2018) for the parameters related to preferences and aggregate output dynamics $\{\beta, \gamma, \psi, \bar{\theta}, \sigma_y, \rho, \sigma_\theta\}$ under objective measure. This allows me to match the mean and the standard deviation of the growth rate of the aggregate output as well as the mean and the standard deviation of annual equity premium and risk-free rate. Given aggregate dividend is more volatile than aggregate output, I report the returns after imposing the financial leverage to be 3 as in Croce (2014).[19]

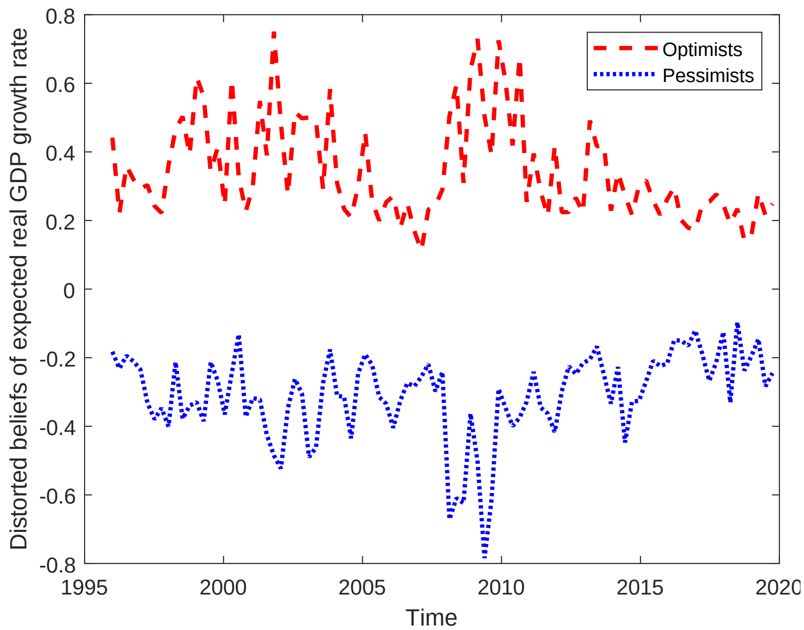
The optimists' and pessimists' long-run mean of growth rate $\{\bar{\theta}^1, \bar{\theta}^2\}$ are calibrated to to match the mean and the standard deviation of optimists' and pessimists' belief deviation relative to the objective measure. I use the forecasts of the annual growth rate of GDP from

[18] As long as the initial Pareto share is not too close to 0 or 1, the quantitative results are similar. This is consistent with the real world, where both optimists and pessimists widely exist. The results of other initial Pareto shares are available upon request.

[19] Ai and Bansal (2018) impose the leverage into the dividend process directly as in Bansal and Yaron (2004), which amplifies the return more than imposing the same leverage after calculating the return on aggregate consumption.

the Survey of Professional Forecasters (SPF).[20]

Figure 7: Beliefs of the expected annual real GDP growth rate



This figure plots the time series of the belief deviations relative to objective measure of the expected annual real GDP growth rate from 1996 to 2019. Data source: Survey of Professional Forecasters (SPF).

I follow Bordalo, Gennaioli, Ma, and Shleifer (2018) to construct the forecast of annual real GDP growth by the professionals from 1996 to 2019. The real GDP growth from end of quarter $t-1$ to end of quarter $t+3$ for each professional i is defined as

$$G^i_{t-1\to t+3} = \frac{F_t x^i_{t+3}}{x^i_{t-1}},$$

where t is the quarter of forecast and x is the level of real GDP is a given quarter; x_{t-1} uses the initial release of actual value in quarter $t-1$, which is available by the time of the forecast in quarter t. $F_t x^i_{t+3}$ is the reported forecast of real GDP growth at the end of quarter $t+3$. I define the distorted beliefs as

$$G^i_{t-1\to t+3} - E\left(G^i_{t-1\to t+3}\right),$$

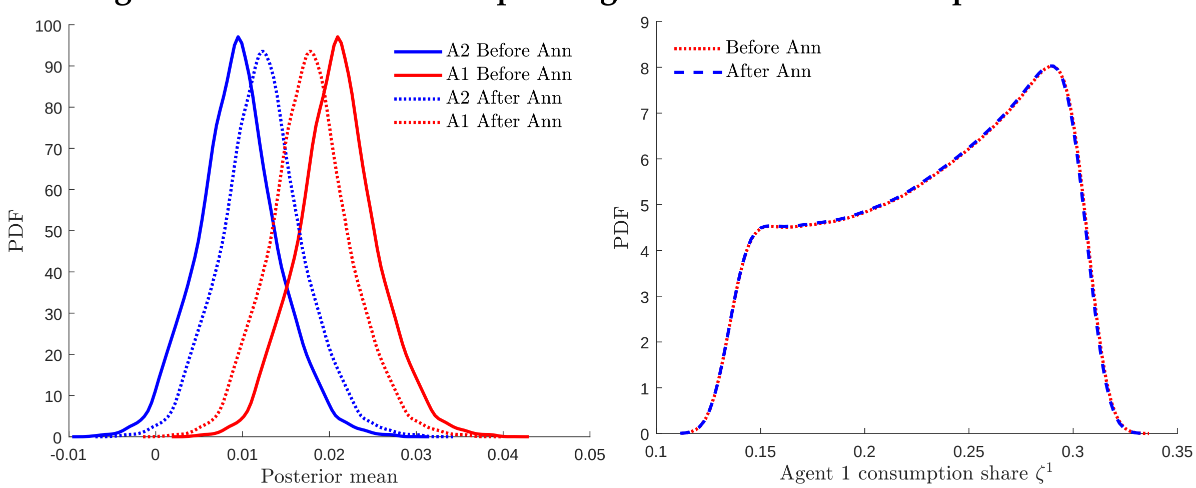
[20]The forecasts of the annual growth rate of GDP from other surveys, such as Blue Chip Economic Indicators, generate smaller magnitudes.

where the $E\left(G^i_{t-1\to t+3}\right)$ is the consensus forecasts. I take the mean and the standard deviation of all positive (negative) belief deviations, as the proxy for the optimists' (pessimists') belief deviations relative to the objective measure. The parameter values are given in Table 2. I report the annual moments in Table 3.

Announcement premium The transparency of announcements (i.e., σ_S^2) plays an essential role in the reduction of uncertainty and disagreement after announcements. As discussed in section 5, this is the main channel to determine the announcement premium after I pin down the optimists' and pessimists' long-run mean of growth rate. Therefore, I calibrate the transparency of announcements to match the equity premium around information releases. Then I study the implications of FOMC announcements on the dynamics of trading volume and heterogeneous beliefs.

6.2 Model implications: trading volume and beliefs around announcements

Figure 8: Distributions of expected growth rate and consumption share

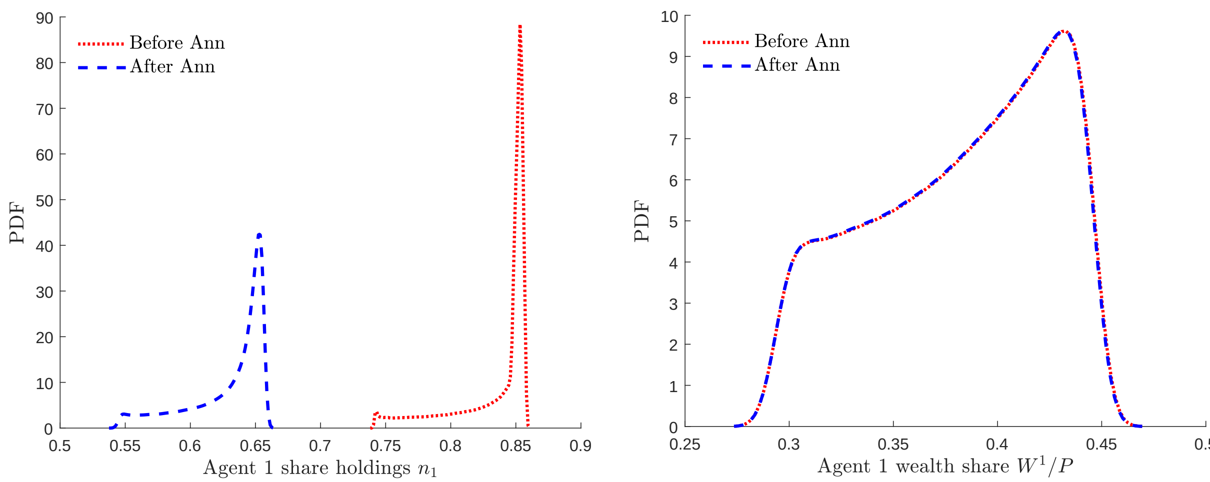


This figure compares the probability densities of expected growth rates and agent 1's consumption share around announcements under objective measure. The parameter values are reported in Table 2.

In this section, I study the implications of key variables related to trading volume and heterogeneous beliefs around announcements in the benchmark calibration. The distributions are approximated by the Monto Carlo simulation.

Figure 8 illustrates the distributions of expected growth rates and agent 1's consumption share before and after announcements under objective measure. The left panel shows, agent 1 has a higher expected growth rate than agent 2. The disagreement becomes smaller after announcements as the dashed lines show. The model implies that the disagreement on growth decreases from 1.15% to 0.55% on announcement days. The right panel shows that the instantaneous reallocation of consumption upon announcements relative to the change of continuation utility is negligible under the benchmark calibration. This is consistent with Figure 4 that the consumption share is not very sensitive to the change of expected growth rate.

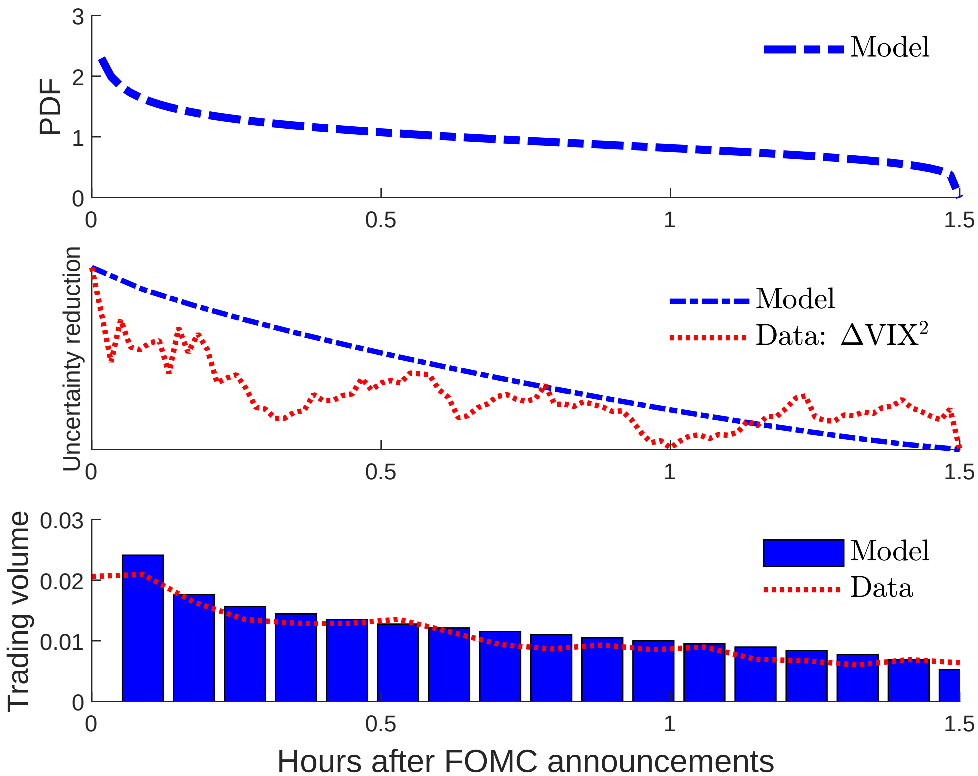
Figure 9: Distributions of asset holdings and wealth share



This figure compares the probability densities of agent 1's asset holdings and wealth share around announcements under objective measure. The parameter values are reported in Table 2.

The left panel of Figure 9 establishes that optimists (the type of agent 1) reduce their shareholdings of the stock significantly since, on average, they become less optimistic after the releases of FOMC news. As defined in equation (28), optimists' shareholdings are determined by two elements: (i) optimists' portfolio share invested in the risky asset π^1; (ii) optimists' wealth share $\frac{W^1}{P}$ in the economy. The right panel of Figure 9 indicts the instantaneous change of wealth share is negligible upon announcements, which implies almost all the trading motivation comes from the first element π^1. Optimists would like to reduce their shareholdings of stock since they become less optimistic (relative to the pessimists) after announcements. Therefore, the unwinding position motivation is the main driver of the associated huge trading volume upon announcements. The reallocation of asset holdings has a significant impact on the wealth accumulation in the long run, which can be interpreted as the information effect on marginal propensity to take risk as defined in Kekre and Lenel (2020).

Figure 10: Model implications: uncertainty reduction and trading volume upon announcements



This figure plots the time series of uncertainty reduction and trading volume of the stock after announcements in the model, given the density of receiving information specified in equation (36). The parameter values and other moments are reported in Table 2 and 3, respectively.

On average, the optimists reduce the stock holdings from 83% to 63% upon FOMC announcements in the benchmark calibration. It implies the average trading volume is 20% out of unwinding positions with less disagreement after the FOMC news.

To capture the time series of trading volume upon announcements, I exogenously specify that the agents in the economy receive the information from a Beta distribution with parameter α, δ on $[0, 1.5]$ hours after announcement. The density of the Beta distribution is

$$f\left(y|\alpha,\delta\right) = B\left[\alpha,\delta\right]^{-1} y^{\alpha-1}\left(1-y\right)^{\delta-1}, \text{ for } y \in \left(0,1\right), \tag{36}$$

where $B\left[\alpha,\delta\right]$ is the Beta function. In my example, the density of receiving information and unwinding positions h hours after announcement is $f\left(1-\frac{h}{1.5}|\alpha,\delta\right)$. I can therefore approximate the uncertainty reduction during hour $(k-h,k)$ as

$$E\left[\int_{k-h}^{k} f\left(1-\frac{t}{1.5}|\alpha,\delta\right)dt\right] \times \Delta\text{Uncertainty}. \tag{37}$$

ΔUncertainty is the average total uncertainty reduction upon announcements that is endogenously determined in the model. I calibrate $\alpha = 0.8$, $\delta = 1.2$ to match the time series of uncertainty reduction after announcements in the data, which is measured by the change of VIX^2. The top panel of Figure 10 shows the density of the Beta distribution, which matches the time series of uncertainty reduction pretty well in the middle panel.

After that, I plot the time series of trading volume during hour $(k-h,k)$, which is approximated by

$$E\left[\int_{k-h}^{k} f\left(1-\frac{t}{1.5}|\alpha,\delta\right)dt\right] \times \left|\int n_{1,T}^{+} d\Phi_T^{+} - \int n_{1,T}^{-} d\Phi_T^{-}\right|, \tag{38}$$

where the total trading volume upon announcements $\left|\int n_{1,T}^{+} d\Phi_T^{+} - \int n_{1,T}^{-} d\Phi_T^{-}\right|$ is endogenously determined in the model. The bottom panel of Figure 10 indicates that the model is consistent with the time series of trading volume after announcements in the data. It captures the rates of the agents receive the FOMC information and decide to unwind positions after announcements.

6.3 The role of recursive preferences

In this section, I compare the quantitative results to CRRA utility to highlight the importance of recursive preferences in the presence of reasonable heterogeneous beliefs. Given $\gamma = 10, \psi = 2$ in the benchmark calibration, I study the following two cases: (1) case I with low

IES $\psi = \frac{1}{\gamma} = \frac{1}{10}$; (2) case II with high IES $\psi = \frac{1}{\gamma} = 2$. To focus on the impact of preferences, I keep all other parameters the same and choose the starting Pareto share so that the average Pareto share over simulations is the same as the benchmark. Table 3 reports the related moments.

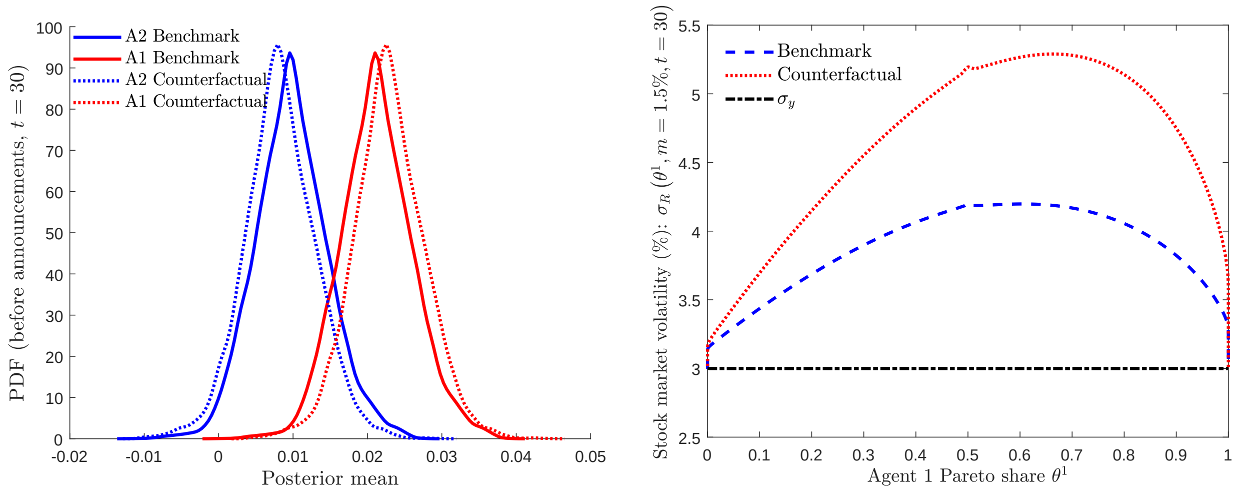
In both cases, while the relative stock holdings change a lot, the agents still trade significantly upon announcements due to the same reduction of disagreement, which implies the dynamics of beliefs are crucial to the trading volume. More importantly, the instantaneous announcement premium is negative in both cases. Note that this violates one of the main results in Ai and Bansal (2018): expected utility will generate zero announcement premium. The main reason is that they focus on a representative-agent model and assume that aggregate consumption does not instantaneously respond to the FOMC announcements.[21] In the framework with heterogeneous agents, though the aggregate consumption does not respond to the news, the reallocation of consumption among agents and the dynamics of beliefs change the SDF upon announcements. Given CRRA utility generates negative premium under reasonable belief dynamics, a more significant generalized risk sensitivity (here recursive preferences with $\gamma > \frac{1}{\psi}$) is necessary for the announcement premium in the framework with heterogeneous beliefs.

6.4 Counterfactual Analysis

The importance of the transparency of FOMC announcements has been discussed for a long time. For example, according to the annual report from Federal Reserve Board in 1923, "The more fully the public understands what the function of the Federal Reserve System is, and on what grounds its policies and actions are based, the simpler and easier will be the problems of credit administration in the U.S." Bernanke (2008) claims the Federal Reserve needs "one more step on the road toward greater transparency". That is the reason he proposes the Chair of the Board of Governors holds a press conference following half of the announcements since April 2011.

[21] Ai, Bansal, Im, and Ying (2019) relax the second assumption that they study a representative-agent model in the production economy, where the aggregate consumption instantaneously responds to the announcements. They also conclude the generalized risk sensitivity is required to account for the announcement premium.

Figure 11: Distributions of beliefs and stock market volatility before announcements

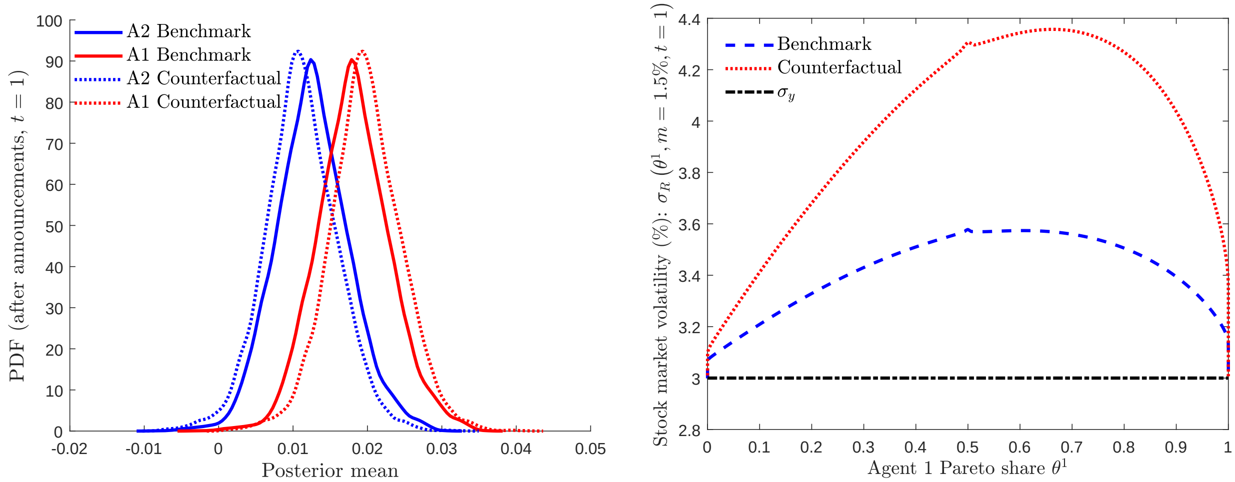


This figure compares the probability densities of expected growth rates and stock market volatility before announcements under objective measure in the counterfactual analysis, where all future announcements' transparency reduces by half.

To highlight the importance of this concern through the consequences of long-run belief divergence and asset market fluctuations, I study a counterfactual economy that the transparency of all future announcements decreases by half. This means the volatility of announcements increases by two times in the model. I find that the average long-run disagreement increases 0.31%: the optimists' expected growth rate increases from 1.95% to 2.11%, while the pessimist' expected growth rate decreases from 1.1% to 0.95%. The implied average stock market volatility is 16.8% higher due to more speculative trading from higher diverged opinions.

The left panel of Figure 11 compares the distribution of both agents' expected growth rates before announcements. The agents in the counterfactual economy have a higher disagreement, which leads to a higher stock market volatility. The same pattern also holds for all other days, including the day just after announcements, as shown in Figure 12. Therefore, my counterfactual analysis quantitatively measures the effects of transparency through long-run belief divergence and the stability of the stock market.

Figure 12: Distributions of beliefs and stock market volatility after announcements



This figure compares the probability densities of expected growth rates and stock market volatility after announcements under objective measure in the counterfactual analysis, where all future announcements' transparency reduces by half.

7 Conclusion

This paper focuses on the effect of FOMC announcements on the dynamics of disagreement and asset market fluctuations. Both call and put open interest of SPX decrease significantly after FOMC announcements, which indicate that the associated high trading volume comes from unwinding positions with less disagreement. Motivated by the asset-market-based evidence, I present a general equilibrium model with heterogeneous beliefs under recursive utility to measure the changes of beliefs out of announcements. The information from the aggregate output and FOMC announcements lead to endogenous disagreement dynamics and time-varying speculative trading. Furthermore, the counterfactual analysis highlights the importance of the transparency of FOMC announcements.

Several remarks are in order. First, the idea of using daily open interest to study whether the disagreement increases or decreases after FOMC announcements can be applied to other high-frequency events, especially when the survey data is not available. Second, this paper concludes that the generalized risk sensitivity is still necessary to account for the significant announcement premium under reasonable disagreement dynamics. This extends the result of Ai and Bansal (2018) that focus on a representative-agent economy. Third, the computational method of the 3-dimensional PDE, moreover, is general enough to be used to solve a wide class of problems, including those faced outside the asset pricing literature.

APPENDICES

The following appendices provide details of the data construction for the empirical evidence in Section 2, the planner's problem in Section 4, the proofs of the main results in Section 5, and the computational method. Appendix A is the data appendix. Appendix B contains all the proofs. Appendix C provides the numerical solutions of the economy with heterogeneous beliefs under recursive utility.

Appendix A Data Description

FOMC announcements There are a total of eight pre-scheduled FOMC meetings each calendar year. The scheduled news release time is taken from the FOMC meeting minutes. For meetings lasting two calendar days, I consider the second day (the day the statement is released) as the event date. Beginning from March 2013, all the FOMC announcements are released at 2:00 PM EST. Before that, the announcements are either released at 2:15 PM EST or 12:30 PM EST.

High-frequency trading volume and return My primary data is comprised of intraday transaction prices and trading volume of E-mini S&P 500 futures (E-mini). In order to mitigate the effect of market micro-structure noise, I follow standard practice in the literature to sparsely sample the data at a 5-minute sampling interval.

Open interest The daily open interest data of SPX comes from Option Metrics starting from the year 2011. To identify option contracts associated with S&P 500 index, I follow WRDS introduction, namely I use those whose secid equals to 108105. Then I separate the contracts to call and put options. I calculate the sum of open interest with respect to all available call (put) contracts on a single day, including different strike prices, and moneyness as long as the maturity is less than 7 days.

Appendix B Proofs

B.1 Proof of Proposition 1

From equation (3),

$$\begin{aligned} dm^i(t) &= \rho\left(\bar{\theta}^i - m^i(t)\right)dt + \frac{Q^i(t)}{\sigma_y^2}\left[\frac{dY_t}{Y_t} - m^i(t)\,dt\right] \\ &\overset{(1)}{=} \rho\left(\bar{\theta}^i - m^i(t)\right)dt + \frac{Q^i(t)}{\sigma_y^2}\left[\left(\theta_t - m^i(t)\right)dt + \sigma_y dB_{Y,t}\right] \\ &= \left[\rho\bar{\theta}^i - \left(\rho + \frac{Q^i(t)}{\sigma_y^2}\right)m^i(t) + \frac{Q^i(t)}{\sigma_y^2}\theta_t\right]dt + \frac{Q^i(t)}{\sigma_y}dB_{Y,t}. \end{aligned} \tag{39}$$

When the agents have the same prior variance as the agent with objective measure, $Q^1(t) = Q^2(t) = Q(t)$ for all t since $Q^i(t)$ is a deterministic function of t as described in equation (4). This implies

$$dm^1(t) - dm^2(t) = \left[\rho\left(\bar{\theta}^1 - \bar{\theta}^2\right) - \left(\rho + \frac{Q(t)}{\sigma_y^2}\right)\left(m^1(t) - m^2(t)\right)\right]dt, \tag{40}$$

which is a first order differential equation. The solution to ODE is

$$m_t^1 - m_t^2 = \rho\left(\bar{\theta}^1 - \bar{\theta}^2\right)e^{A(t)}\int_0^t e^{-A(s)}ds + e^{-A(t)}D \tag{41}$$

with $A(t) = \int_0^t \left(\rho + \frac{Q(u)}{\sigma_y^2}\right)du$, where $D = m_0^1 - m_0^2$ is the initial disagreement. In addition, $Q(t)$ obeys a Riccati equation. One can easily show that $Q(t)$ has a closed-form solution. In general, I have [22]

$$Q^i(t) = \frac{\sigma_\theta^2\left(1 - e^{-2\hat{\rho}(t+t^*)}\right)}{(\hat{\rho}-\rho)\,e^{-2\hat{\rho}(t+t^*)} + \rho + \hat{\rho}}, \tag{42}$$

where t^* is defined as:

$$t^* = \frac{1}{2\hat{\rho}}\ln\frac{\sigma_\theta^2 + (\hat{\rho}-\rho)\,Q^i(0)}{\sigma_\theta^2 - (\hat{\rho}+\rho)\,Q^i(0)}.$$

Upon announcements, from equation (8),

$$\begin{aligned} m_{nT}^{1,+} - m_{nT}^{2,+} &= Q_{nT}^+\left[\frac{1}{\sigma_S^2}s_n + \frac{1}{Q_{nT}^-}m_{nT}^{1,-}\right] - Q_{nT}^+\left[\frac{1}{\sigma_S^2}s_n + \frac{1}{Q_{nT}^-}m_{nT}^{2,-}\right] \\ &= \frac{Q_{nT}^+}{Q_{nT}^-}\left(m_{nT}^{1,-} - m_{nT}^{2,-}\right) < m_{nT}^{1,-} - m_{nT}^{2,-}. \end{aligned} \tag{43}$$

Furthermore, combining equations (41) and (43), I can calculate the initial disagreement D in the stationary economy through

$$D = \frac{Q_T^+}{Q_T^-}\left[\rho\left(\bar{\theta}^1 - \bar{\theta}^2\right)e^{A(T)}\int_0^T e^{-A(s)}ds + e^{-A(T)}D\right],$$

which implies the initial disagreement $D = \frac{\frac{Q_T^+}{Q_T^-}\rho\left(\bar{\theta}^1 - \bar{\theta}^2\right)e^{A(T)}\int_0^T e^{-A(s)}ds}{1 - \frac{Q_T^+}{Q_T^-}e^{-A(T)}}$. The initial disagreement is zero when the announcement fully reveals the information. It's straightforward to show when the agents have the same long run growth rate ($\bar{\theta}^1 = \bar{\theta}^2$), the disagreements will vanish in the long run. Similarly, I can calculate the belief distortions relative to the objective measure through equation (39).

[22] With $Q(0) = 0$, I have

$$Q(t) = \frac{\sigma_\theta^2\left(1 - e^{-2\hat{\rho}t}\right)}{(\hat{\rho}-\rho)\,e^{-2\hat{\rho}t} + \rho + \hat{\rho}}$$

where $\hat{\rho}$ is defined as:

$$\hat{\rho} = \sqrt{\rho^2 + \sigma_\theta^2/\sigma_y^2}$$

and $Q(t) \to Q^* = \frac{\sigma_\theta^2}{\rho + \hat{\rho}}$ as $t \to \infty$.

B.2 Proof of Proposition 2

From the definition of the return

$$dR_t = \frac{Y_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} d\tilde{B}_{Y,t} = \mu^i_{R,t} dt + \sigma^i_{R,t} d\tilde{B}^i_{Y,t}$$

and the equivalence of the measures (6),

$$\sigma_{R,t} \equiv \sigma^i_{R,t},\ \mu^i_{R,t} = \mu_{R,t} + \bar{\mu}^i(t)\,\sigma_{R,t} \quad t \in ((n-1)T, nT)$$

Thus, the agents have the same perceived volatility of returns and their perceived return is captured by their differences of opinion about the expected endowment growth rate.

B.3 Proof of Proposition 3

Applying Ito's lemma to λ^1_t leads to a maximization problem subjective measure,

$$\lambda^1_t V^1_t\left(C^1\right) = \sup_{\{\nu^1_s\}_{s\geq t}} E_t\left[\int_t^\infty \lambda^1_s F\left(C^1_s,\, \nu^1_s\right) ds\right] \tag{44}$$

subject to

$$d\lambda^1_t = \lambda^1_t\left[-\nu^1_t dt + \bar{\mu}^1(t)\, d\tilde{B}_{Y,t}\right],\ \lambda^1_0 > 0 \text{ given } \bar{\mu}^1(0) \tag{45}$$

I can do the same thing for agent 2. Therefore, I can write the social planner under objective measure, as equation (21) states.

Since $J_0\left(\lambda^1_0, \lambda^2_0, Y_0, m_0\right) = \sup_{(C^1,C^2,\nu^1,\nu^2)} \left\{E_0\left[\int_0^\infty \lambda^1_s F\left(C^1_s,\, \nu^1_s\right) ds\right] + E_0\left[\int_0^\infty \lambda^2_s F\left(C^2_s,\, \nu^2_s\right) ds\right]\right\}$, the corresponding HJB equation is

$$\begin{aligned}\sup_{(C^1,C^2,\nu^1,\nu^2)} \lambda^1_t F\left(C^1_t,\, \nu^1_t\right) + \lambda^2_t F\left(C^2_t,\, \nu^2_t\right) + E_t\left\{J_t dt + J_{\lambda^1} d\lambda^1 + J_{\lambda^2} d\lambda^2 + J_Y dY + J_m dm\right\} \\ + \frac{1}{2} E_t\left\{J_{\lambda^1\lambda^1}\left(d\lambda^1\right)^2 + J_{\lambda^2\lambda^2}\left(d\lambda^2\right)^2 + J_{YY} dY^2 + J_{mm} dm^2\right\} + \\ E_t\left\{J_{\lambda^1\lambda^2} d\lambda^1 d\lambda^2 + J_{\lambda^1 Y} d\lambda^1 dY + J_{\lambda^1 m} d\lambda^1 dm + J_{\lambda^2 Y} d\lambda^2 dY + J_{\lambda^2 m} d\lambda^2 dm + J_{Ym} dY dm\right\} = 0\end{aligned} \tag{46}$$

By the law of motion of the state variables, we can simply the above HJB equation,

$$\begin{aligned}\sup_{(C^1,C^2,\nu^1,\nu^2)} \lambda^1_t F\left(C^1_t,\, \nu^1_t\right) + \lambda^2_t F\left(C^2_t,\, \nu^2_t\right) + J_t - \nu^1_t \lambda^1_t J_{\lambda^1} - \nu^2_t \lambda^2_t J_{\lambda^2} + m_t Y_t J_Y + \rho\left(\bar{\theta} - m(t)\right) J_m \\ + \frac{1}{2} J_{\lambda^1\lambda^1}\left(\nu^1_t \lambda^1_t\right)^2 + J_{\lambda^2\lambda^2}\left(\nu^2_t \lambda^2_t\right)^2 + J_{YY}\sigma_y^2 Y_t^2 + J_{mm}\frac{Q^2(t)}{\sigma_y} + \\ + J_{\lambda^1 Y}\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)\lambda^1_t \sigma_y Y_t + J_{\lambda^1 m}\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)\lambda^1_t \frac{Q(t)}{\sigma_y} + J_{Ym} Q(t) = 0.\end{aligned} \tag{47}$$

The maximization over $\left(\nu^1, \nu^2\right)$ of the HJB equation can be solved separately since $\left(\nu^1, \nu^2\right)$ only appears in the first term for both agents. Define

$$f\left(C^i, J_{\lambda^i}, t\right) \doteq \sup_{\nu^i} F\left(C_t^i, \nu_t^i\right) - \nu_t^i J_{\lambda^i} = \frac{\beta}{1-\frac{1}{\psi}} \left\{ \left(C_t^i\right)^{1-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^i}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} - (1-\gamma) J_{\lambda^i} \right\}, \tag{48}$$

and the first order condition with respect to ν^i shows

$$\nu^i = \frac{\beta}{1-\frac{1}{\psi}} \left[1 - \gamma - \left(\frac{1}{\psi} - \gamma\right) \left(C_t^i\right)^{1-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^i}\right]^{\frac{\frac{1}{\psi}-1}{1-\gamma}} \right].$$

I denote the Lagrange multiplier on the market clearing as ξ_{MC}. The first order condition with respect to C^i of the HJB (47) is

$$\lambda_t^i \beta \left(C_t^i\right)^{-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^i}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \xi_{MC} = 0, \quad i = 1, 2$$

which means

$$\lambda_t^1 \left(C_t^1\right)^{-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^1}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} = \lambda_t^2 \left(C_t^2\right)^{-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^2}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}}$$

Therefore, I can derive the consumption share $\zeta^i = \frac{C^i}{Y}$ for each agent i as,

$$\zeta^i = \frac{\lambda_t^i \left(C_t^i\right)^{-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^i}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}}}{\lambda_t^1 \left(C_t^1\right)^{-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^1}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \lambda_t^2 \left(C_t^2\right)^{-\frac{1}{\psi}} \left[(1-\gamma) J_{\lambda^2}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}}} \tag{49}$$

Due to the homogeneity of the value function,

$$J\left(\lambda_1, \lambda_2, Y, m, t\right) = Y^{1-\gamma} \left(\lambda_1 + \lambda_2\right) \tilde{J}\left(\frac{\lambda_1}{\lambda_1 + \lambda_2}, m, t\right) = Y^{1-\gamma} \theta_2 \tilde{J}\left(\varpi^1, m, t\right)$$

where $\varpi^1 = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $\varpi^2 = \lambda_1 + \lambda_2$. ϖ^1 represents the Pareto share of agent 1, which is obviously bounded between zero and one.

I apply Ito's Lemma to $J\left(\lambda_1, \lambda_2, Y, m, t\right)$ to derive all the elements in equation (47). Combing with equation (48), I can simply the HJB equation as

$$
\begin{aligned}
0 = {} & \varpi^1 \frac{\beta}{1-\frac{1}{\psi}} \left(\zeta^1\right)^{1-\frac{1}{\psi}} \left((1-\gamma)\tilde{J}^1\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \left(1-\varpi^1\right) \frac{\beta}{1-\frac{1}{\psi}} \left(1-\zeta^1\right)^{1-\frac{1}{\psi}} \left((1-\gamma)\tilde{J}^2\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} \\
& + \tilde{J}_t + \left(-\frac{\beta(1-\gamma)}{1-\frac{1}{\psi}} + m(1-\gamma) + \left(\varpi^1 \bar{\mu}^1(t) + \left(1-\varpi^1\right)\bar{\mu}^2(t)\right)(1-\gamma)\sigma_y - \frac{1}{2}\gamma(1-\gamma)\sigma_y^2\right)\tilde{J} \\
& + \varpi^1\left(1-\varpi^1\right)\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)(1-\gamma)\sigma_y \tilde{J}_{\varpi^1} + \frac{1}{2}\left(1-\varpi^1\right)^2\left(\varpi^1\right)^2\left(\bar{\mu}^1(t)-\bar{\mu}^2(t)\right)^2 \tilde{J}_{\varpi^1\varpi^1} \\
& + \left[\rho\left(\bar{\theta}-m\right) + (1-\gamma)Q(t) + \left(\varpi^1\bar{\mu}^1(t) + \left(1-\varpi^1\right)\bar{\mu}^2(t)\right)\frac{Q(t)}{\sigma_y}\right]\tilde{J}_m + \\
& \varpi^1\left(1-\varpi^1\right)\left(\bar{\mu}^1(t)-\bar{\mu}^2(t)\right)\frac{Q(t)}{\sigma_y}\tilde{J}_{\varpi^1 m} + \frac{1}{2}\left(\frac{Q(t)}{\sigma_y}\right)^2 \tilde{J}_{mm}
\end{aligned}
\tag{50}
$$

The functions $\tilde{J}^n\left(\varpi^1, m, t\right)$ are the continuation values of two agents scaled by $Y^{1-\gamma}$,

$$
\begin{aligned}
\tilde{J}^1\left(\varpi^1, m, t\right) &\doteq \tilde{J}\left(\varpi^1, m, t\right) + \left(1-\varpi^1\right)\tilde{J}_{\varpi^1}\left(\varpi^1, m, t\right) \\
\tilde{J}^2\left(\varpi^1, m, t\right) &\doteq \tilde{J}\left(\varpi^1, m, t\right) - \varpi^1 \tilde{J}_{\varpi^1}\left(\varpi^1, m, t\right)
\end{aligned}
$$

and the consumption share ζ^1 is given

$$
\zeta^1\left(\varpi^1, m, t\right) = \frac{\left(\varpi^1\right)^{\psi}\left[(1-\gamma)\tilde{J}^1\left(\varpi^1, m, t\right)\right]^{\frac{1-\psi\gamma}{1-\gamma}}}{\left(\varpi^1\right)^{\psi}\left[(1-\gamma)\tilde{J}^1\left(\varpi^1, m, t\right)\right]^{\frac{1-\psi\gamma}{1-\gamma}} + \left(1-\varpi^1\right)^{\psi}\left[(1-\gamma)\tilde{J}^2\left(\varpi^1, m, t\right)\right]^{\frac{1-\psi\gamma}{1-\gamma}}}
$$

and agent 2's consumption share $\zeta^2\left(\varpi^1, m, t\right) = 1 - \zeta^1\left(\varpi^1, m, t\right).$

Now the boundary condition. Let $\varpi^1 = 0$ or 1, I can derive the following boundary conditions of ϖ^1: $\tilde{J}(0, m, t) = H^2(m, t)$ and $\tilde{J}(1, m, t) = H^1(m, t)$, where $H^n(m, t)$ are the solution to the following equation

$$
\begin{aligned}
0 = {} & \frac{\beta}{1-\frac{1}{\psi}}\left[(1-\gamma)H\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + H_t + \left[-\frac{\beta(1-\gamma)}{1-\frac{1}{\psi}} + (1-\gamma)\left(m + \sigma_y \bar{\Pi}^i(t)\right) - \frac{1}{2}\gamma(1-\gamma)\sigma_y^2\right] H \\
& + \left[\rho\left(\bar{\theta}-m\right) + (1-\gamma)Q(t) + \bar{\Pi}^i(t)\frac{Q(t)}{\sigma_y}\right] H_m + \frac{1}{2}\frac{Q^2(t)}{\sigma_y^2} H_{mm},
\end{aligned}
\tag{51}
$$

where $\bar{\Pi}^i(t) = \bar{\mu}^i(t)$.

The law of motion of agent 1's Pareto share ϖ^1 follows

$$
\begin{aligned}
d\varpi^{1} &= \frac{\lambda_{2}}{\left(\lambda_{1}+\lambda_{2}\right)^{2}} d\lambda_{1} - \frac{\lambda_{1}}{\left(\lambda_{1}+\lambda_{2}\right)^{2}} d\lambda_{2} - \frac{\lambda_{2}}{\left(\lambda_{1}+\lambda_{2}\right)^{3}} d\lambda_{1}^{2} + \frac{\lambda_{1}}{\left(\lambda_{1}+\lambda_{2}\right)^{3}} d\lambda_{2}^{2} + \frac{2\left(\lambda_{1}-\lambda_{2}\right)}{\left(\lambda_{1}+\lambda_{2}\right)^{3}} d\lambda_{1} d\lambda_{2} \\
&= \varpi^{1}\left(1-\varpi^{1}\right)\left[\lambda_{t}^{2}-\lambda_{t}^{1}+\left(\varpi^{1}\bar{\mu}^{1}(t)+\left(1-\varpi^{1}\right)\bar{\mu}^{2}(t)\right)\left(\bar{\mu}^{2}(t)-\bar{\mu}^{1}(t)\right)\right] dt \\
&+\varpi^{1}\left(1-\varpi^{1}\right)\left(\bar{\mu}^{1}(t)-\bar{\mu}^{2}(t)\right) d\tilde{B}_{Y,t} \\
&\doteq \mu_{\varpi^{1}}\left(\varpi^{1}, m, t\right) dt + \sigma_{\varpi^{1}}\left(\varpi^{1}, m, t\right) d\tilde{B}_{Y,t}
\end{aligned}
$$

As long as the drift of ϖ^1 is not always positive or negative, for strictly positive initial weights, the boundaries are unattainable, so that ϖ^1 evolves on the open interval $(0,1)$. Therefore, both agents can survive in the long run as shown in Borovicka (2018).

B.4 The dynamics of wealth accumulation and stock return

Due to the homogeneity of the recursive preference, aggregate wealth in units of consumption at time t must be

$$
W_{t}^{1} = \frac{(1-\gamma) V_{t}^{1}}{D_{C} f\left(C_{t}^{1}, V_{t}^{1}\right)} = \frac{1}{\beta} Y_{t}\left[\zeta^{1}\left(\varpi^{1}, m, t\right)\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^{1}\left(\varpi^{1}, m, t\right)\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \tag{52}
$$

Define $H\left(\varpi^{1}, m, t\right) = \left[\zeta^{1}\left(\varpi^{1}, m, t\right)\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^{1}\left(\varpi^{1}, m, t\right)\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$ and apply the Ito's Lemma to the above equation, I can derive the following law of motion

$$
\begin{aligned}
\frac{dW_{t}^{1}}{W_{t}^{1}} &= \frac{dY_{t}}{Y_{t}} + \frac{H_{t}}{H} dt + \frac{H_{\varpi^{1}}}{H} d\varpi^{1} + \frac{1}{2}\frac{H_{\varpi^{1}\varpi^{1}}}{H}\left(d\varpi^{1}\right)^{2} + \frac{H_{m}}{H} dm \\
&+\frac{1}{2}\frac{H_{mm}}{H}(dm)^{2} + \frac{H_{\varpi^{1}m}}{H} d\varpi^{1} dm + \frac{H_{\varpi^{1}}}{H} d\varpi^{1}\frac{dY_{t}}{Y_{t}} + \frac{H_{m}}{H} dm \frac{dY_{t}}{Y_{t}} \\
&= \left[m_{t} + \frac{H_{t}}{H} + \frac{H_{\varpi^{1}}}{H}\mu_{\theta} + \frac{H_{m}}{H}\mu_{m} + \frac{1}{2}\frac{H_{\varpi^{1}\varpi^{1}}}{H}\sigma_{\theta}^{2} + \frac{1}{2}\frac{H_{mm}}{H}\sigma_{m}^{2} + \frac{H_{\varpi^{1}m}}{H}\sigma_{\theta}\sigma_{m} + \frac{H_{\varpi^{1}}}{H}\sigma_{\theta}\sigma_{y} + \frac{H_{m}}{H}\sigma_{m}\sigma_{y}\right] dt \\
&+\left[\sigma_{y} + \frac{H_{\varpi^{1}}}{H}\sigma_{\theta} + \frac{H_{m}}{H}\sigma_{m}\right] d\tilde{B}_{Y,t} \\
&= \left[m_{t} + \frac{H_{t}}{H} + \frac{H_{\varpi^{1}}}{H}\left(\mu_{\theta}+\sigma_{\theta}\sigma_{y}\right) + \frac{H_{m}}{H}\left(\mu_{m}+\sigma_{m}\sigma_{y}\right) + \frac{1}{2}\frac{H_{\varpi^{1}\varpi^{1}}}{H}\sigma_{\theta}^{2} + \frac{1}{2}\frac{H_{mm}}{H}\sigma_{m}^{2} + \frac{H_{\varpi^{1}m}}{H}\sigma_{\theta}\sigma_{m}\right] dt \\
&+\left[\sigma_{y} + \frac{H_{\varpi^{1}}}{H}\sigma_{\theta} + \frac{H_{m}}{H}\sigma_{m}\right] d\tilde{B}_{Y,t} \\
&\doteq \mu_{W^{1}}\left(\varpi^{1}, m, t\right) dt + \sigma_{W^{1}}\left(\varpi^{1}, m, t\right) d\tilde{B}_{Y,t} \\
&= \left[\mu_{W^{1}}\left(\varpi^{1}, m, t\right) + \bar{\mu}^{1}(t)\sigma_{W^{1}}\left(\varpi^{1}, m, t\right)\right] dt + \sigma_{W^{1}}\left(\varpi^{1}, m, t\right) d\tilde{B}_{Y,t}^{1}
\end{aligned}
$$

Similarly, I define $G\left(\varpi^1,m,t\right)=\left[\zeta^2\left(\varpi^1,m,t\right)\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^2\left(\varpi^1,m,t\right)\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$ and drive the law of motion for agent 2's wealth accumulation W_t^2,

$$\begin{aligned}
\frac{dW_t^2}{W_t^2}&=\left[m_t+\frac{G_t}{G}+\frac{G_{\varpi^1}}{G}\left(\mu_\theta+\sigma_\theta\sigma_y\right)+\frac{G_m}{G}\left(\mu_m+\sigma_m\sigma_y\right)+\frac{1}{2}\frac{G_{\varpi^1\varpi^1}}{G}\sigma_\theta^2+\frac{1}{2}\frac{G_{mm}}{G}\sigma_m^2+\frac{G_{\varpi^1 m}}{G}\sigma_\theta\sigma_m\right]dt\\
&\quad+\left[\sigma_y+\frac{G_{\varpi^1}}{G}\sigma_\theta+\frac{G_m}{G}\sigma_m\right]d\tilde{B}_{Y,t}\\
&\doteq\mu_{W^2}\left(\varpi^1,m,t\right)dt+\sigma_{W^2}\left(\varpi^1,m,t\right)d\tilde{B}_{Y,t}\\
&=\left[\mu_{W^2}\left(\varpi^1,m,t\right)+\bar{\mu}^2(t)\,\sigma_{W^2}\left(\varpi^1,m,t\right)\right]dt+\sigma_{W^2}\left(\varpi^1,m,t\right)d\tilde{B}_{Y,t}^2.
\end{aligned}$$

Given the law of motions of both agents' wealth accumulation, I can derive the process for stock return. Since bonds are in zero net supply, the wealth of the two agents must sum to the value of the risky asset, that is

$$W_t=W_t^1+W_t^2=P_t.$$

This implies the stock return follows,

$$\begin{aligned}
dR_t=\frac{Y_tdt+dW_t}{W_t}&=\frac{Y_tdt+dW_t}{W_t}=\frac{Y_t}{W_t^1+W_t^2}dt+\frac{W_t^1}{W_t^1+W_t^2}\frac{dW_t^1}{W_t^1}+\frac{W_t^2}{W_t^1+W_t^2}\frac{dW_t^2}{W_t^2}\\
&=\frac{\beta+\left[\zeta^1\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^1\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\mu_{W^1}+\left[\zeta^2\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^2\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\mu_{W^2}}{\left[\zeta^1\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^1\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}+\left[\zeta^2\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^2\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}dt\\
&\quad+\frac{\left[\zeta^1\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^1\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\sigma_{W^1}+\left[\zeta^2\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^2\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\sigma_{W^2}}{\left[\zeta^1\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^1\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}+\left[\zeta^2\right]^{\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^2\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}d\tilde{B}_{Y,t}\\
&\doteq\mu_R\left(\varpi^1,m,t\right)dt+\sigma_R\left(\varpi^1,m,t\right)d\tilde{B}_{Y,t} \qquad (53)\\
&=\left[\mu_R\left(\varpi^1,m,t\right)+\bar{\mu}^i(t)\,\sigma_R\left(\varpi^1,m,t\right)\right]dt+\sigma_R\left(\varpi^1,m,t\right)d\tilde{B}_{Y,t}^i. \qquad (54)
\end{aligned}$$

where the last line shows the return process under agent i's subjective measure.

Recall that the dynamics of agent i's wealth under his subjective measure follows:

$$\begin{aligned}
\frac{dW_t^i}{W_t^i}&=\left[r_t+\pi_t^i\left(\mu_{R,t}^i-r_t\right)-\frac{C_t^i}{W_t^i}\right]dt+\pi_t^i\sigma_R\left(\varpi^1,m,t\right)d\tilde{B}_{Y,t}^i\\
&\doteq\mu_{W^i}\left(\varpi^1,m,t\right)dt+\sigma_{W^i}\left(\varpi^1,m,t\right)d\tilde{B}_{Y,t}^i.
\end{aligned}$$

$\pi^i\left(\varpi^1,m,t\right)$ is the agent i's portfolio share invested in the risky asset at period t and is determined by

$$\pi^i\left(\varpi^1,m,t\right)=\frac{\sigma_{W^i}\left(\varpi^1,m,t\right)}{\sigma_R\left(\varpi^1,m,t\right)},$$

where the stock market volatility $\sigma_R\left(\varpi^1, m, t\right)$ is defined in equation (54).

B.5 The dynamics of SDF

Since agent 2's SDF under his subjective belief is given by:

$$\begin{aligned}\Lambda^2 &= \exp\left(-\int_0^t \nu_s^2 ds\right) D_C F\left(C_t^2, \nu_s^2\right) \\ &= \beta \exp\left(-\int_0^t \nu^2\left(\varpi^1, m, s\right) ds\right) Y_t^{-\gamma}\left[\zeta^2\left(\varpi^1, m, t\right)\right]^{-\frac{1}{\psi}}\left[(1-\gamma)\tilde{J}^2\left(\varpi^1, m, t\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}, \end{aligned} \tag{55}$$

I apply Ito's Lemma to derive the law of motion with respect to his perspective:

$$\begin{aligned}\frac{d\Lambda^2}{\Lambda^2} &= \Lambda_t^2 dt - \gamma\frac{dY_t}{Y_t} + \frac{1}{2}\gamma(\gamma+1)\left(\frac{dY_t}{Y_t}\right)^2 + \frac{\Lambda_1^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}d\varpi^1 + \frac{1}{2}\frac{\Lambda_{11}^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}\left(d\varpi^1\right)^2 + \frac{\Lambda_2^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}dm \\ &+ \frac{1}{2}\frac{\Lambda_{22}^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}(dm)^2 + \frac{\Lambda_{12}^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}d\varpi^1 dm + \left(-\gamma\frac{\Lambda_1^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}\right)d\varpi^1\frac{dY_t}{Y_t} + \left(-\gamma\frac{\Lambda_2^2\left(\varpi^1, m, t\right)}{\Lambda^2\left(\varpi^1, m, t\right)}\right)dm\frac{dY_t}{Y_t} \\ &= \left[\Lambda_t^2 - \gamma m + \frac{1}{2}\gamma(\gamma+1)\sigma_y^2 + \frac{\Lambda_1^2}{\Lambda^2}\left(\mu_\theta - \gamma\sigma_\theta\sigma_y\right) + \frac{\Lambda_2^2}{\Lambda^2}\left(\mu_m - \gamma\sigma_m\sigma_y\right) + \frac{1}{2}\frac{\Lambda_{11}^2}{\Lambda^2}\sigma_\theta^2 + \frac{1}{2}\frac{\Lambda_{22}^2}{\Lambda^2}\sigma_m^2 + \frac{\Lambda_{12}^2}{\Lambda^2}\sigma_m\sigma_\theta\right]dt + \\ &\quad \left[-\gamma\sigma_y + \frac{\Lambda_1^2}{\Lambda^2}\sigma_\theta + \frac{\Lambda_2^2}{\Lambda^2}\sigma_m\right]d\tilde{B}_{Y,t} \\ &\doteq -r\left(\varpi^1, m, t\right)dt - \sigma_\Lambda^2\left(\varpi^1, m, t\right)d\tilde{B}_{Y,t}^2\end{aligned}$$

Therefore, the risk free rate is

$$\begin{aligned} r\left(\varpi^1, m, t\right) &= -\Lambda_t^2 + \sigma_\Lambda^2\left(\varpi^1, m, t\right)\bar{\mu}^2(t) + \gamma m - \frac{1}{2}\gamma(\gamma+1)\sigma_y^2 \\ &- \frac{\Lambda_{\varpi^1}^2}{\Lambda^2}\left(\mu_\theta - \gamma\sigma_\theta\sigma_y\right) - \frac{\Lambda_m^2}{\Lambda^2}\left(\mu_m - \gamma\sigma_m\sigma_y\right) - \frac{1}{2}\frac{\Lambda_{\varpi^1\varpi^1}^2}{\Lambda^2}\sigma_\theta^2 - \frac{1}{2}\frac{\Lambda_{mm}^2}{\Lambda^2}\sigma_m^2 - \frac{\Lambda_{\varpi^1 m}^2}{\Lambda^2}\sigma_m\sigma_\theta, \end{aligned} \tag{56}$$

and the perceived market price of risk by agent 2 is determined by

$$\begin{aligned}\sigma_\Lambda^2\left(\varpi^1, m, t\right) &= \gamma\sigma_y - \frac{1}{\psi}\left[\frac{\zeta_{\varpi^1}^2\left(\varpi^1, m, t\right)}{\zeta^2\left(\varpi^1, m, t\right)}\sigma_{\varpi^1} + \frac{\zeta_m^2\left(\varpi^1, m, t\right)}{\zeta^2\left(\varpi^1, m, t\right)}\sigma_m\right] \\ &- \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left[\frac{\tilde{J}_{\varpi^1}^2\left(\varpi^1, m, t\right)}{\tilde{J}^2\left(\varpi^1, m, t\right)}\sigma_{\varpi^1} + \frac{\tilde{J}_m^2\left(\varpi^1, m, t\right)}{\tilde{J}^2\left(\varpi^1, m, t\right)}\sigma_m\right].\end{aligned} \tag{57}$$

Similarity, agent 1's SDF under his subjective measure is given by

$$\frac{d\Lambda^1}{\Lambda^1} = -r\left(\varpi^1, m, t\right)dt - \sigma_\Lambda^1\left(\varpi^1, m, t\right)d\tilde{B}_{Y,t}^1$$

where

$$\sigma_{\Lambda}^{1}\left(\varpi^{1},m,t\right)=\gamma\sigma_{y}-\frac{1}{\psi}\left[\frac{\zeta_{\varpi^{1}}^{1}\left(\varpi^{1},m,t\right)}{\zeta^{1}\left(\varpi^{1},m,t\right)}\sigma_{\varpi^{1}}+\frac{\zeta_{m}^{1}\left(\varpi^{1},m,t\right)}{\zeta^{1}\left(\varpi^{1},m,t\right)}\sigma_{m}\right]$$
$$-\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left[\frac{\tilde{J}_{\varpi^{1}}^{1}\left(\varpi^{1},m,t\right)}{\tilde{J}^{1}\left(\varpi^{1},m,t\right)}\sigma_{\varpi^{1}}+\frac{\tilde{J}_{m}^{1}\left(\varpi^{1},m,t\right)}{\tilde{J}^{1}\left(\varpi^{1},m,t\right)}\sigma_{m}\right]. \tag{58}$$

Besides, the perceived market price of risk between the two types of agents is linked by

$$\sigma_{\Lambda}^{2}\left(\varpi^{1},m,t\right)=\sigma_{\Lambda}^{1}\left(\varpi^{1},m,t\right)+\bar{\mu}^{2}\left(t\right)-\bar{\mu}^{1}\left(t\right). \tag{59}$$

Appendix C The computational method

I solve the partial differential equation (PDE) in (22) with a finite difference method that approximates the function $\tilde{J}\left(\varpi^1, m, t\right)$ on a three-dimensional grid, where $\varpi^1 \in \left\{\varpi_i^1\right\}_{i=1}^I$, $m \in \left\{m_j\right\}_{j=1}^J$ and $t = \{1, 2, ...30\}$. The distance between the grid points on m is $\triangle m$. The grid on $\varpi^1 \in [0,1]$ is non-uniform to capture the high curvature of value function with respect to ϖ^1. I use the notation $\tilde{J}_{i,j}^t \equiv \tilde{J}\left(\varpi_i^1, m_j, t\right)$.

I approximate the first derivatives of $\tilde{J}$ using both backward and forward differences and second derivatives with central differences. Denoting by $\varpi_{i+1}^1 - \varpi_i^1 = \triangle\varpi_{i,+}^1$, $\varpi_i^1 - \varpi_{i-1}^1 = \triangle\varpi_{i,-}^1$, the forward and backward distance between two grid points of ϖ_i^1, the derivatives are evaluated according to

$$
\begin{aligned}
\frac{\partial \tilde{J}_{i,j}^{t+1}}{\partial \varpi^1} &\equiv \left(\tilde{J}_{\varpi^1}^F\right)_{i,j}^{t+1} \approx \frac{\tilde{J}_{i+1,j}^{t+1} - \tilde{J}_{i,j}^{t+1}}{\triangle\varpi_{i,+}^1},\\
\frac{\partial \tilde{J}_{i,j}^{t+1}}{\partial \varpi^1} &\equiv \left(\tilde{J}_{\varpi^1}^B\right)_{i,j}^{t+1} \approx \frac{\tilde{J}_{i,j}^{t+1} - \tilde{J}_{i-1,j}^{t+1}}{\triangle\varpi_{i,-}^1},\\
\frac{\partial \tilde{J}_{i,j}^{t+1}}{\partial m} &\equiv \left(\tilde{J}_{m}^F\right)_{i,j}^{t+1} \approx \frac{\tilde{J}_{i,j+1}^{t+1} - \tilde{J}_{i,j}^{t+1}}{\triangle m},\\
\frac{\partial \tilde{J}_{i,j}^{t+1}}{\partial m} &\equiv \left(\tilde{J}_{m}^B\right)_{i,j}^{t+1} \approx \frac{\tilde{J}_{i,j}^{t+1} - \tilde{J}_{i,j-1}^{t+1}}{\triangle m},\\
\frac{\partial^2 \tilde{J}_{i,j}^{t+1}}{\partial \left(\varpi^1\right)^2} &= \left(\tilde{J}_{\varpi^1\varpi^1}\right)_{i,j}^{t+1} \approx \frac{\triangle\varpi_{i,-}^1 \tilde{J}_{i+1,j}^{t+1} - \left(\triangle\varpi_{i,-}^1 + \triangle\varpi_{i,+}^1\right)\tilde{J}_{i,j}^{t+1} + \triangle\varpi_{i,+}^1 \tilde{J}_{i-1,j}^{t+1}}{\frac{1}{2}\left(\triangle\varpi_{i,-}^1 + \triangle\varpi_{i,+}^1\right)\triangle\varpi_{i,-}^1\triangle\varpi_{i,+}^1},\\
\frac{\partial^2 \tilde{J}_{i,j}^{t+1}}{\partial m^2} &= \left(\tilde{J}_{mm}\right)_{i,j}^{t+1} \approx \frac{\tilde{J}_{i,j+1}^{t+1} - 2\tilde{J}_{i,j}^{t+1} + \tilde{J}_{i,j-1}^{t+1}}{\left(\triangle m\right)^2},\\
\frac{\partial^2 \tilde{J}_{i,j}^{t+1}}{\partial \varpi^1 \partial m} &= \left(\tilde{J}_{\varpi^1 m}\right)_{i,j}^{t+1} \approx \frac{\tilde{J}_{i+1,j+1}^{t+1} - \tilde{J}_{i+1,j-1}^{t+1} - \tilde{J}_{i-1,j+1}^{t+1} + \tilde{J}_{i-1,j-1}^{t+1}}{2\left(\triangle\varpi_{i,-}^1 + \triangle\varpi_{i,+}^1\right)\triangle m},
\end{aligned}
\tag{60}
$$

where the choice of forward or backward derivatives depends on the sign of the drift function for the state variable.

The first step is to solve the boundary conditions (23) through the finite difference method. When $\varpi^1 = 1$, equation (23) is approximated by the following upwind scheme under $\triangle t = \frac{1}{360}$ to capture the daily frequency:

$$
\begin{aligned}
&\frac{H_j^{1,t+1} - H_j^{1,t}}{\triangle t} + w_j H_j^{1,t+1} = U_j^t + v_j\left(H_m^{1,t+1}\right)_j + \frac{1}{2}\frac{Q^2(t)}{\sigma_y^2}\left(H_{mm}^{1,t+1}\right)_j\\
&= U_j^t + v_j^+ \frac{H_{j+1}^{1,t+1} - H_j^{1,t+1}}{\triangle m} + v_j^- \frac{H_j^{1,t+1} - H_{j-1}^{1,t+1}}{\triangle m} + \frac{1}{2}\frac{Q^2(t)}{\sigma_y^2}\frac{H_{j+1}^{1,t+1} - 2H_j^{1,t+1} + H_{j-1}^{1,t+1}}{\left(\triangle m\right)^2}
\end{aligned}
\tag{61}
$$

where

$$w_j = \frac{\beta(1-\gamma)}{1-\frac{1}{\psi}} - (1-\gamma)\left(m_j + \sigma_y \bar{u}^1(t)\right) + \frac{1}{2}\gamma(1-\gamma)\sigma_y^2,$$
$$v_j = \rho\left(\bar{\theta} - m_j\right) + (1-\gamma)Q(t) + \bar{u}^1(t)\frac{Q(t)}{\sigma_y},$$
$$U_j^t = \frac{\beta}{1-\frac{1}{\psi}}\left[(1-\gamma)H_j^{1,t}\right]^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}}.$$

Then I update the pre-announcement value function $H^1(m,T)$ using

$$H^1\left(m^-,T^-\right) = E_T^-\left[H^1\left(m^+,T^+\right)\right].$$

Go back to equation (61) until the function $H^1(m,T)$ converges.[23] Similarly, I solve the boundaries when $\varpi^1 = 0$—$H^2(m,T)$.

After I derive the boundary conditions, I approximate the HJB equation (22) using the derivatives from equations (60):

$$\begin{aligned}
&\frac{\tilde{J}_{i,j}^{t+1} - \tilde{J}_{i,j}^t}{\triangle t} + \left(\frac{\beta(1-\gamma)}{1-\frac{1}{\psi}} - m_j(1-\gamma) - \left(\varpi_i^1\bar{\mu}^1(t) + \left(1-\varpi_i^1\right)\bar{\mu}^2(t)\right)(1-\gamma)\sigma_y + \frac{1}{2}\gamma(1-\gamma)\sigma_y^2\right)\tilde{J}_{i,j}^{t+1} \\
&= \varpi_i^1\frac{\beta}{1-\frac{1}{\psi}}\left(\zeta_{i,j}^1\right)^{1-\frac{1}{\psi}}\left((1-\gamma)\tilde{J}_{i,j}^{1,t}\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \left(1-\varpi_i^1\right)\frac{\beta}{1-\frac{1}{\psi}}\left(1-\zeta_{i,j}^1\right)^{1-\frac{1}{\psi}}\left((1-\gamma)\tilde{J}_{i,j}^{2,t}\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} \\
&\quad + \varpi_i^1\left(1-\varpi_i^1\right)\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)(1-\gamma)\sigma_y\left(\tilde{J}_{\varpi^1}\right)_{i,j}^{t+1} + \\
&\quad \left[\rho\left(\bar{\theta} - m_j\right) + (1-\gamma)Q(t) + \left(\varpi_i^1\bar{\mu}^1(t) + \left(1-\varpi_i^1\right)\bar{\mu}^2(t)\right)\frac{Q(t)}{\sigma_y}\right]\left(\tilde{J}_m\right)_{i,j}^{t+1} \\
&\quad + \varpi_i^1\left(1-\varpi_i^1\right)\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)\frac{Q(t)}{\sigma_y}\left(\tilde{J}_{\varpi^1 m}\right)_{i,j}^{t+1} \\
&\quad + \frac{1}{2}\left(1-\varpi_i^1\right)^2\left(\varpi_i^1\right)^2\left(\bar{\mu}^1(t) - \bar{\mu}^2(t)\right)^2\tilde{J}_{\varpi^1\varpi^1}^{t+1} + \frac{1}{2}\left(\frac{Q(t)}{\sigma_y}\right)^2\left(\tilde{J}_{mm}\right)_{i,j}^{t+1}.
\end{aligned} \tag{62}$$

The above equations can be written in matrix notation as

$$\frac{1}{\triangle t}\left(\tilde{J}^{t+1} - \tilde{J}^t\right) + w^t\tilde{J}^{t+1} = U^t + \mathbb{C}\tilde{J}^{t+1}. \tag{63}$$

After I use the backward induction to calculate $\tilde{J}(\varpi^1, m, T^+)$, I update the pre-announcement value function $\tilde{J}(\varpi^1, m, T^-)$ using

$$\tilde{J}\left(\varpi^1, m^-, T^-\right) = E_T^-\left[\tilde{J}\left(\varpi^1, m^+, T^+\right)\right].$$

[23] The boundary on m is reflected since m_t follows the standard Ornstein–Uhlenbeck process.

Go back to equation (62) until the function $\tilde{J}\left(\varpi^1, m, T\right)$ converges. I impose the boundary conditions $\tilde{J}\left(0, m, T\right) \equiv H^2\left(m, T\right)$ and $\tilde{J}\left(1, m, T\right) \equiv H^1\left(m, T\right)$ during the iterations.

Table 1: The change of open interest of SPX around FOMC announcements

This table reports the change of open interest of SPX comparing to last day around FOMC announcements. To measure the instantaneous effects of announcements, I focus on the options with the maturity less than 7 days. I also report the result of the same calculation on all other three-day window that do not contain FOMC announcements. ***Significant at 1%, **Significant at 5%, *Significant at 10%. I report the t-statistics using the day-clustered standard error in parenthesis.

	Calls Change (%)		Puts Change (%)	
	Day 0	Day 1	Day 0	Day 1
FOMC announcements	-53.5***	-11.0	-47.1***	6.1
	(-3.909)	(-0.728)	(-4.184)	(0.520)
Other three-day window	2.0	0.7	1.7	0.1
	(0.997)	(0.362)	(0.952)	(0.055)

Table 2: Parameters

The model is calibrated at annually frequency. I assume the prescheduled announcements happen at the monthly frequency, that is, $T = \frac{1}{12}$.

Parameter	symbol	value
aggregate output		
long run output growth rate	$\bar{\theta}$	1.50%
volatility of output	σ_y	3%
persistence of the AR(1) process	ρ	10%
volatility of the AR(1) process	σ_θ	0.25%
disagreements and uncertainty		
optimists' long-run mean of growth rate	$\bar{\theta}^1$	0.4099
pessimists' long-run mean of growth rate	$\bar{\theta}^2$	-0.3400
the transparency of announcements	σ_S^2	8.5×10^{-7}
preference		
risk aversion	γ	10
elasticity of intertemporal substitution	ψ	2
subjective discount factor	β	0.02

Table 3: Moments: macro quantities, asset prices, and asset holdings

This table presents macro quantities, asset prices, and asset holdings from the data, and three cases from the models: the benchmark ($\gamma = 10, \psi = 2$), CRRA I ($\psi = \frac{1}{\gamma} = \frac{1}{10}$) and CRRA II ($\psi = \frac{1}{\gamma} = 2$). Panel A reports the belief distortions relative to the objective measure. Panel B reports the annual macro moments and asset prices under objective measure. I impose the financial leverage of 3 when I calculate the moments related to equity premium. Panel C reports other implications of the model.

Moments	Data	Model Benchmark $\gamma = 10, \psi = 2$	Model CRRA I $\psi = \frac{1}{\gamma} = \frac{1}{10}$	Model CRRA II $\psi = \frac{1}{\gamma} = 2$
Panel A: *Heterogeneous beliefs*				
Mean of optimists' belief deviation	0.45%	0.45%	0.45%	0.45%
Mean of pessimists' belief deviation	-0.40%	-0.40%	-0.40%	-0.40%
Std of optimists' belief deviation	0.095%	0.095%	0.095%	0.095%
Std of pessimists' belief deviation	0.088%	0.086%	0.086%	0.086%
Panel B: *Macro quantities and asset prices*				
Mean of agg. consumption growth	1.5%	1.44%	1.44%	1.44%
Std of agg. output growth	2.5%	2.43%	2.43%	2.43%
Average risk-free rate	0.40%	1.75%	12.85%	2.39%
Std of risk-free rate	2.85%	0.17%	3.19%	0.16%
Annual equity premium	6.06%	4.78%	2.85%	0.10%
Std of annual equity premium	19.8%	9.55%	32.17%	9.36%
Instantaneous FOMC ann. premium	3.8 *bps*	3.4 *bps*	-15.0 *bps*	-0.71*bps*
Panel C: *Other implications*				
Stock holdings of optimists before ann.		0.83	1.32	2.88
Stock holdings of optimists after ann.		0.63	0.76	1.61
Mean of optimists' consumption share		23.1%	53.9%	95.8%
Std of optimists' consumption share		5.2%	0.44%	1.38%

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