The Pre-FOMC Announcement Drift and Private Information: Kyle Meets Macro-Finance

Chao Ying*

Abstract

This paper studies the private information explanation for the timing and time series pattern of the pre-FOMC announcement drift. I document informed trading is in the same direction as the realized returns in the 24-hour window before FOMC. I extend Kyle's (1985) model to be the case where market makers are compensated for the riskiness of assets' fundamentals. Observing aggregate order flow, market makers update their belief about the marginal-utility-weighted asset value, which gradually resolves uncertainty, resulting in an upward drift in market prices. I demonstrate a strictly positive pre-FOMC announcement drift if and only if market makers require risk compensation.

Keywords: Pre-FOMC announcement drift; informed trading; private information; risk compensation; uncertainty resolution

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*Chao Ying (`chaoying@cuhk.edu.hk`) is affiliated with Department of Finance, the Chinese University of Hong Kong. I am grateful to Robert Goldstein (Chair), Xiaoji Lin, Erzo Luttmer, and Colin Ward for their continuous guidance and encouragement. I thank Hengjie Ai, Kerry Back (discussant), Luca Benzoni, Markus Brunnermeier, Anna Cieslak, Julio Crego (discussant), Jeffrey Cross (discussant), Philip Dybvig, Michal Dzielinski, Murray Frank, Lorenzo Garlappi, Jungsuk Han (discussant), Ramin Hassan, Yunzhi Hu, Shohini Kundu (discussant), Jangwoo Lee, Moritz Lenel, Tao Li, Erik Loualiche, Liang Ma (discussant), Gustavo Manso, Dmitriy Muravyev, Pavel Savor (discussant), Neil Pearson, Jose Penalva, Lin Peng, Walter Pohl, Martin Szydlowski, Xiaoxiao Tang (discussant), Stathis Tompaidis (discussant), Vish Viswanathan (discussant), Tracy Wang, Yajun Wang (discussant), Michael Weber (discussant), Wei Wu, Ram Yamarthy, Terry Zhang, and participants at the Stanford SITE Asset Pricing Session, 4th SAFE Market Microstructure Conference, Future of Financial Information Webinar Series, Virtual Derivatives Workshop, NFA Doctoral Symposium, OFR PhD Symposium, FMA Doctoral Consortium and Presentation Session, Young Economist Symposium, PhD - Economics Virtual Seminar, WashU EGSC, UCLA SOCAE, Econometric Society European Winter Meeting, European Winter Finance Conference, FIRS, Econometric Society Summer Meeting North America, World Symposium on Investment Research, China International Risk Forum, EFA 2021, Peking University HSBC, Texas A&M University, Tsinghua University SEM, Tsinghua University PBC, CKGSB, Boston University, Renmin University Hanqing, Tulane University, Rice University, Chinese University of Hong Kong, University of Southern California, City University of Hong Kong, and University of Minnesota for their helpful comments.

1 Introduction

Lucca and Moench (2015) document substantial stock market returns before the Federal Open Market Committee (FOMC) announcements. They find that the pre-FOMC announcement drift of the S&P 500 index is 49 basis points on average during the 24-hour window preceding FOMC announcements, which corresponds to about 80% of the annual realized excess returns in the stock market. However, the hours and days before FOMC meetings fall into a blackout period, a time when policymakers and Fed staff refrain from discussions of monetary policy information.[1] It presents a notable challenge to standard asset pricing theory, which predicts that equity returns should be earned *at*, *rather than ahead of*, the announcements when uncertainty is resolved from public news .

Some papers have offered suggestive evidence that the pre-FOMC announcement drift may come from private information before announcements. Cieslak, Morse, and Vissing-Jorgensen (2019) and Vissing-Jorgensen (2020) provide a history of leaked discussions in FOMC documents and argue that systematic information leakage drives the pre-FOMC announcement drift. In addition to information leakage, market participants may generate their proprietary information by collecting data related to FOMC announcements.[2] In this paper, I study the private information explanation for the timing and time series pattern of the pre-FOMC announcement drift through informed trading.

Empirically, I provide asset-market-based evidence that supports the presence of private information and informed trading before FOMC announcements. First, Hu, Pan, Wang, and Zhu (2020) find a significant and systematic reduction of market uncertainty (measured by the CBOE VIX index) during the same 24-hour window before FOMC announcements. Second, sorting the FOMC days into terciles via a 24-hour uncertainty reduction before announcements, I find that the only group with a substantial uncertainty reduction preceding announcements is associated with a positive pre-FOMC announcement drift. Third, to measure informed trading, I calculate the order imbalances, defined as the difference between buyer- and seller-initiated trading volumes divided by total trading volume. When a reduction in uncertainty occurs before FOMC announcements, the abnormal order imbal-

[1] The blackout period begins at the start of the second Saturday midnight ET before the beginning of the meeting and ends at midnight ET on the next day after the meeting.

[2] For example, Kurov, Sancetta, Strasser, and Wolfe (2017) show that proprietary information permits forecasting announcement surprises in some cases.

ances are 1.85%-2.17% higher in the direction of the realized return in the 24-hour window before FOMC announcements.

To understand the above features of the financial markets, I build a model in which the pre-FOMC announcement drift is earned as risk is reduced through information released from informed trading. I integrate Kyle's (1985) model into an endowment economy with learning such that market makers are compensated for the riskiness of assets' fundamentals. The equity premium is realized with an uncertainty reduction prior to announcements since insider trading reveals private information. Characterizing the equilibrium price and insider trading by a closed-form method, I establish a strictly positive pre-FOMC announcement drift if and only if market makers are *risk compensated*.

The aggregate economic growth rate is driven by a latent state variable that is unobservable to all investors but periodically announced by the Federal Reserve. Since FOMC announcements provide information about the macroeconomy, market makers require risk compensation in assets' fundamentals before announcements and set the price equal to the marginal-utility-weighted payoffs. The countercyclical stochastic discount factor (SDF) applies extra discounting to payoffs that are positively correlated with utility. The insider knows the underlying information before announcements and trades to maximize the expected terminal profit, understanding that the trading affects the price. Meanwhile, noise traders have random, price-inelastic demands, as in the standard Kyle model. By observing aggregate order flow, market markers update the estimation of asset payoffs as well as the SDF simultaneously such that uncertainty is resolved before FOMC announcements.

Here are some implications of the equilibrium with risk-compensated market makers. First, the equilibrium price is a submartingale instead of a martingale in standard continuous-time Kyle-type models.[3] Because of risk compensation, the price of risky assets increases on average as uncertainty is resolved through insider trading before announcements. The slope of the expected pre-FOMC announcement drift is the negative covariance between the innovation of the SDF and the asset value. I prove a strictly positive pre-FOMC announcement drift if and only if market makers are compensated for the riskiness of assets' fundamentals. The positive excess return leads to positive average order imbalances before announcements. In the meantime, to entice the insider to trade and release

[3]See, among others, Back (1992), Back and Pedersen (1998), Li (2013), and Collin-Dufresne and Fos (2016).

information early, market makers have incentives to set a price impact that increases on average, implying the submartingale property of the price impact. Second, because of the average upward drift in market prices, market makers rationally anticipate that the insider would trade positively on average in order to chase that premium. The insider also has to consider the additional price impact from uncertainty resolution via her trading, which is unique in this model. In the equilibrium, instead of being zero, the expected order rate is determined by the announcement premium per unit of time to Kyle's lambda. Additionally, as market makers converge to be risk-neutral, the limit of the equilibrium is well defined and converges to the traditional Kyle model. The equilibrium implications indicate that this paper provides a microfoundation for how the diffusion of private news drives positive pre-FOMC announcement drift in a standard microstructure framework.

To account for the timing and time series pattern of the pre-FOMC announcement drift, I extend the benchmark in the following two directions. First, the insider knows private information earlier than 24 hours before FOMC announcements and chooses a starting time that will maximize expected profits. Increasing uncertainty before announcements means that the insider wants to trade later when the market is noisier. She cannot trade too late, however, since she needs substantial liquidity trading in order to hide her position. Owing to the trade-off between uncertainty and liquidity, I find in the calibration that informed profits are highest when the starting time is 24 hours before announcements, which explains the timing of the pre-FOMC announcement drift. Second, since uncertainty is not always reduced before FOMC meetings, I generalize the benchmark that the insider may not be better informed, and market makers assess whether or not the insider has private information. In addition to updating their belief of asset payoffs and the SDF, market makers estimate the probability that insiders have private information simultaneously. The pricing rule is nonlinear and stochastic, which drives the price volatility, market depth, and price response to be stochastic. The model can quantitatively explain the time series pattern of the pre-FOMC announcement drift and the uncertainty reduction as well as informed trading.

Before concluding, I demonstrate that other forms of asset market evidence around FOMC announcements are consistent with the model's predictions. First, I document that since April 2011, market uncertainty has decreased significantly only before announcements with press conferences, which explains the two distinctive patterns in equity returns found in Boguth, Gregoire, and Martineau (2019) and consistent with the full model. Second, the sign of the pre-FOMC announcement drift in the model depends on whether the asset is risky or a hedge. The average of the time-varying betas of nominal bonds is close to zero from 1996 to 2019, resulting in the absence of a pre-FOMC announcement drift in fixed income instruments. Third, the pre-FOMC announcement drift is empirically stronger when there is a greater reduction in uncertainty before announcements, consistent with the risk-based explanation.

Related literature

The paper relates to several strands of the literature. First is the large body of work investigating the impact of asymmetric information on asset prices and price impacts, seminal examples of which include Kyle (1985) and Back (1992).[4] I build on this literature by exploring the implications of risk-compensated market makers. Though Subrahmanyam (1991), Çetin and Danilova (2016), and Back, Cocquemas, Ekren, and Lioui (2021) study risk-averse market makers, none of them can generate the positive pre-FOMC announcement drift because they focus on inventory risk as in Stoll (1978), and the return depends on the initial inventory. Since people can trade through the highly liquid S&P 500 E-mini futures, the corresponding inventory risk is negligible before FOMC announcements. In my framework, market makers are compensated for the riskiness of assets' fundamentals, which is revealed following FOMC news. The equilibrium price in this model is a submartingale instead of a martingale since the resolution of uncertainty is associated with realizations of premiums, in contrast to the models in the literature.

My paper theoretically contributes to the literature on the premium around FOMC announcements. Hu, Pan, Wang, and Zhu (2020) and Laarits (2019) contribute the pre-FOMC announcement drift to an uncertainty reduction in a representative agent framework. Cocoma (2020) constructs a general equilibrium model of disagreement where two groups of investors react differently to announcements. Ai and Bansal (2018) provide a revealed preference theory for the macroeconomic an-

[4] A short list of this literature is Foster and Viswanathan (1996), Back and Pedersen (1998), Back, Cao, and Willard (2000), Caldentey and Stacchetti (2010), Li (2013), Collin-Dufresne and Fos (2016), Back, Crotty, and Li (2018), Drechsler, Moreira, and Savov (2018), Dai, Wang, and Yang (2019), and Crego (2020).

nouncement premium in a representative agent economy. Ai, Bansal, and Han (2021) apply the same preference and study the pre-FOMC announcement drift where investors have different incentives to acquire public information. My paper mainly differs in two aspects. First, I show a strictly positive pre-FOMC announcement drift if and only if market makers require risk compensation instead of the generalized risk sensitivity stated in Ai and Bansal (2018). Second, this model endogenously explains the timing and time series pattern of the pre-FOMC announcement drift, which helps us understand how the information is revealed before FOMC announcements.

This paper builds on the literature of the macroeconomic announcement premium. Savor and Wilson (2013) find a significant equity market return on days with major macroeconomic announcements.[5] Lucca and Moench (2015) document the substantial stock market return during the 24-hour period preceding FOMC announcements.[6] Hu, Pan, Wang, and Zhu (2020) find that market uncertainty decreases in the 24-hour window before FOMC announcements, which is consistent with the private information explanation proposed by Cieslak, Morse, and Vissing-Jorgensen (2019) and Vissing-Jorgensen (2020). This paper provides additional empirical support from the classifications of FOMC announcements and order imbalances, which motivates the theoretical framework.

The rest of the paper is organized as follows. Section 2 provides empirical evidence that indicates the presence of private information before FOMC news. Section 3 extends Kyle's (1985) model to the case in which market makers are risk compensated and characterizes the equilibrium price and insider trading. In section 4, I generalize the benchmark model to account for the timing and time series pattern of the pre-FOMC announcement drift. Section 5 tests the further implications of the model. Section 6 concludes. The Appendix contains additional details on the empirical analysis as well as the proofs.

[5]Ernst, Gilbert, and Hrdlicka (2019) find that FOMC announcements, which include the largest point estimates for the concentration of the equity premium, appear to stand out from other macroeconomic announcements. Giacoletti, Ramcharan, and Yu (2020) study the impact of FOMC announcements on the mortgage market.

[6]Some papers argue that the low realized volatility before FOMC announcements rules out the private information explanation. However, FOMC announcement days may be accompanied by different types of risks compared to other days, and only part of that risk may be revealed before FOMC announcements. In addition, speculation of the FOMC news may increase the volatility of noise traders before announcements, even though these traders do not have private information, which also can lead to the low realized volatility.

2 Empirical Evidence

In this section, I summarize the potential sources of private information before FOMC announcements and discuss the suggestive evidence shown in the literature. After that, I provide asset-market-based evidence that supports the presence of private information before FOMC news. First, I present that market uncertainty (measured by the VIX index) decreases significantly and systematically during the same window as the pre-FOMC announcement drift, as documented in Hu, Pan, Wang, and Zhu (2020). Second, sorting the FOMC days into terciles via a 24-hour uncertainty reduction before announcements, I find that the only group with a substantial uncertainty reduction preceding announcements is associated with a positive pre-FOMC announcement drift. Third, I document that there is significant insider trading (measured by order imbalances) only when uncertainty decreases before announcements, which is consistent with the private information explanation.

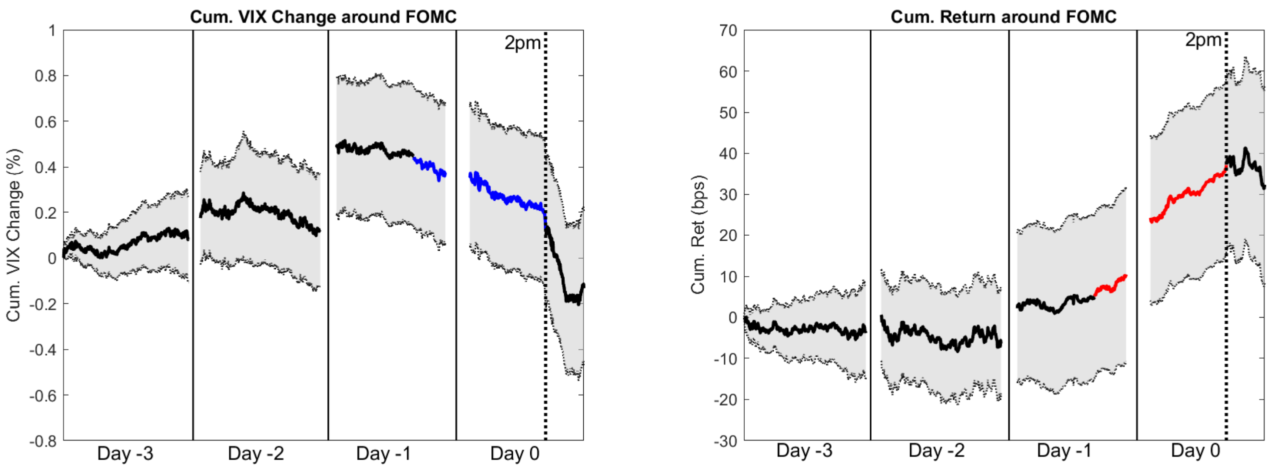
2.1 Sources of private information before FOMC meetings

The literature has provided suggestive evidence of private information before FOMC announcements. The private information may be obtained by leakage. Cieslak, Morse, and Vissing-Jorgensen (2019) propose that information about the Federal Reserve's unexpected accommodating monetary policy is leaked ahead of the FOMC announcement, which causes a pre-announcement equity market rally. Vissing-Jorgensen (2020) provides a history of leaked discussions in FOMC documents to show that the FOMC itself expresses frequent concerns about leaks. For example, the leakage led to the resignation of Jeffrey Lacker, president of the Federal Reserve Bank of Richmond, following an admission of his involvement in the leak of confidential FOMC information to Medley Global Advisors in 2012. Finer (2018) documents an abnormal number of new York City taxi rides to the district of Liberty Street at certain times before FOMC announcements. Additionally, disclosure of private information may come from accidental information leakage—a word-of-mouth interpretation of information diffusion, which has been well studied in the literature of takeovers (see Keown and Pinkerton (1981), Jarrell and Poulsen (1989), Meulbroek (1992), and Augustin et al. (2015)).

The other potential source of private information is through proprietary data collection related to FOMC announcements. Given the huge market attention to FOMC announcements, to infer what the Federal Reserve knows, institutional investors have a strong motivation to obtain the information that the Federal Reserve observes and keep updating their prediction models of monetary policy from historical data.[7] Kurov, Sancetta, Strasser, and Wolfe (2017) support this explanation by finding that proprietary information permits the forecasting of announcement surprises in some cases.

2.2 The average cumulative VIX change and return before FOMC announcements

Figure 1: The average cumulative VIX change and return around FOMC announcements



This figure shows the average cumulative VIX change ($\Delta\text{VIX}_t = \text{VIX}_t - \text{VIX}_{-3}$ where VIX_{-3} is the initial VIX level at the beginning of Day -3) and average cumulative return on the S&P 500 index within four-day windows from 1996 to 2019. The solid line in the left (right) panel is the average cumulative VIX change (average cumulative return of the SPX) from 9:30 a.m. ET three days prior to scheduled FOMC announcements to 4:00 p.m. ET on days with scheduled FOMC announcements (labeled as Day 0). The blue (red) solid line indicates the VIX change (cumulative return of the SPX) within the 2 p.m. to 2 p.m. pre-FOMC window. The gray shaded areas are pointwise 95% confidence bands around the average. The sample period is from January 1996 to December 2019. The dashed vertical line is set at 2:00 p.m. ET, when FOMC announcements are typically just released or 15 minutes before the release.

To capture the changes in market expectations in a timely manner, I use the CBOE VIX index, which is a model-free measure of implied volatility computed from the S&P 500 index option prices. For the intraday returns, I obtain transaction-level data on the S&P 500 index (SPX). The sample period is from January 1996 to December 2019. During this period, there are 187 scheduled releases of FOMC statements in total. Except for 9 of them, other releases are scheduled around either 2:15 p.m. ET

[7]For example, institutional investors can hire talented, well-trained economists who help the Federal Reserve process and interpret all the information being released, as discussed in Nakamura and Steinsson (2018).

(before April 2011) or 2:00 p.m. ET (after April 2011).[8] Therefore, I follow Lucca and Moench (2015) and focus on the 2 p.m. to 2 p.m. pre-FOMC window, which should not contain any announcement information if no private information is revealed.

Figure 1 shows the average cumulative VIX change and average cumulative return on the S&P 500 index around FOMC announcements. The solid line in the right panel represents the mean pointwise cumulative intraday percentage return of the SPX over a four-day window from the market open of the day ahead of scheduled FOMC meetings to the day after. During the window from Day -3 through the beginning of Day -1, the average VIX increases because of the huge uncertainty surrounding the upcoming FOMC news. As shown in Table 1, the reduction in VIX during the 24-hour period preceding FOMC announcements is 0.3% with a t-stat of -3.4 , which is consistent with Hu, Pan, Wang, and Zhu (2020). Meanwhile, the cumulative pre-FOMC announcement drift over the same window is 33.2 basis points on average, which is statistically significant at the 1% level.

The significant reduction in the VIX index shows the systemic uncertainty reduction following FOMC news being revealed preceding announcements. FOMC members commonly express their views about macroeconomic developments or monetary policy issues in meetings or conversations with members of the public, but they refrain from these discussions in the week before FOMC meetings. The 24-hour pre-FOMC window is part of the blackout period. Therefore, the significant uncertainty reduction in this window indicates the potential presence of private information before FOMC announcements.

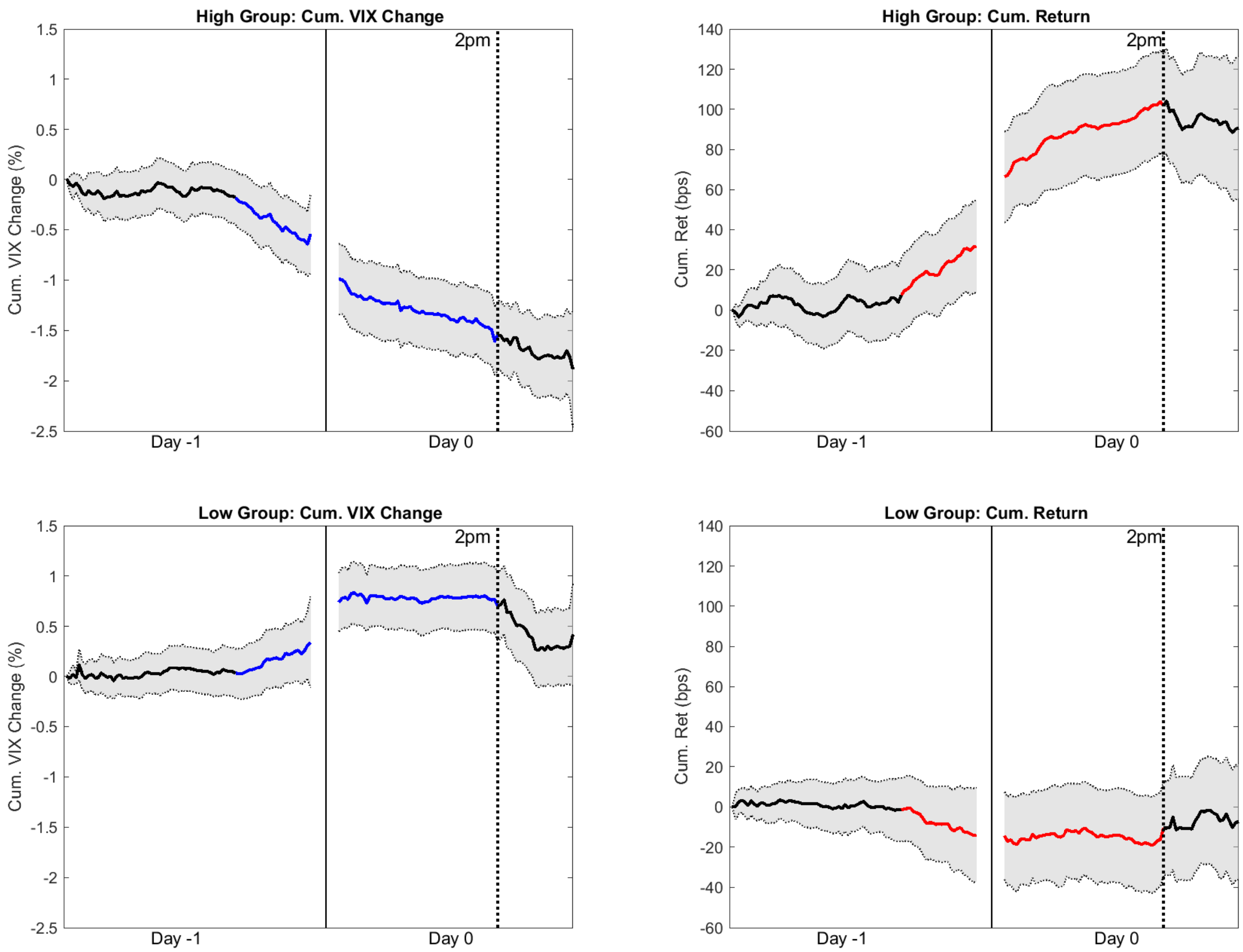
2.3 Classification of FOMC announcements via uncertainty reduction

I sort the FOMC days into terciles by their reduction in uncertainty during the 24-hour window before announcements. Figure 2 plots the cumulative VIX change and the cumulative return around FOMC meetings for the high-reduction and low-reduction groups, separately. The reduction in high-reduction group's VIX index is a significant 1.459% over the 2 p.m. to 2 p.m. pre-FOMC window, which is associated with a deeper pre-FOMC announcement drift (94.4 basis points) relative to the av-

[8] Eight of the 9 exceptions are released around 12:30 p.m. ET from April 2011 to December 2012. Another exception happened at 11:30 a.m. ET on March 26, 1996, because of the Federal Reserve chairman's other duties. The results hold robustly without these releases.

erage FOMC results, as shown in Table 1. By contrast, the low-reduction group's VIX index increases instead of decreases before announcements, and there is no positive pre-announcement drift.

Figure 2: Classifications of FOMC announcements: sort on uncertainty reduction



This figure shows the average cumulative return on the S&P 500 index on two-day windows for the high group and low group, respectively, where I sort the reduction in uncertainty within the 24-hour pre-FOMC window into terciles. The blue (red) solid line indicates the VIX change (cumulative return of the SPX) on the 2 p.m. to 2 p.m. pre-FOMC window.

This classification demonstrates that not all FOMC announcements are the same—only the ones with an uncertainty reduction preceding announcements are associated with a positive pre-FOMC announcement drift. Later I show that this is consistent with the full model in section 4.2 showing that the pre-announcement drift occurs only when the insider is informed.

In addition to the uncertainty reduction prior to FOMC announcements, Abdi and Wu (2018) find that corporate bond returns and trade directions before FOMC announcements predict the pre-FOMC stock market returns. Park (2019) shows that speculators' spread trades in bond futures have predictive information about future FOMC meetings and concludes that private information plays a key role in explaining the pre-FOMC announcement drift. All of these forms of asset market evidence indicate the presence of private information before FOMC news.

2.4 Informed trading before FOMC announcements

Following the microstructure literature, I measure informed trading activity by the order imbalance in the testing security defined as $\frac{B-S}{B+S}$, where B (S) is the aggregate buyer-initiated (seller-initiated) trading volume.[9] I use two measures of imbalance, OIN and OID, where volume is defined as the number of trades and dollar trading volume, respectively.

Following Bernile, Hu, and Tang (2016), I classify the trading volume of E-mini S&P 500 futures (E-mini) as buyer or seller initiated using the tick rule. Specifically, a transaction is classified as buyer initiated (seller initiated) if the transaction price is higher (lower) than the last different transaction price. For each time window, the corresponding order imbalance is the difference between the total buyer- and seller-initiated volumes divided by the total trading volume.

To study the pre-FOMC announcement drift, I examine the 24-hour window preceding FOMC announcements, $[-24H, 0]$. Informed trading leads to the diffusion of private information and the uncertainty reduction before announcements. Therefore, for each announcement, I construct a categorical variable, UR, that equals positive one (negative one) when uncertainty is reduced, and the cumulative return is positive (negative) over the 24-hour window. UR is zero otherwise.

In Figure 3, for each FOMC announcement, I plot the order imbalance based on the number of trades (OIN) and the dollar volume (OID) in the 24-hour window before FOMC. When there is an uncertainty reduction ($UR = \pm 1$), most order imbalances tend to be in the direction of the realized return before announcements and are large in magnitude. When uncertainty does not decrease before announcements ($UR = 0$), however, the order imbalance is smaller and largely random. Table

[9] Ahern (2020) finds that order imbalance is one of the most robust predictors of insider trading after all controls.

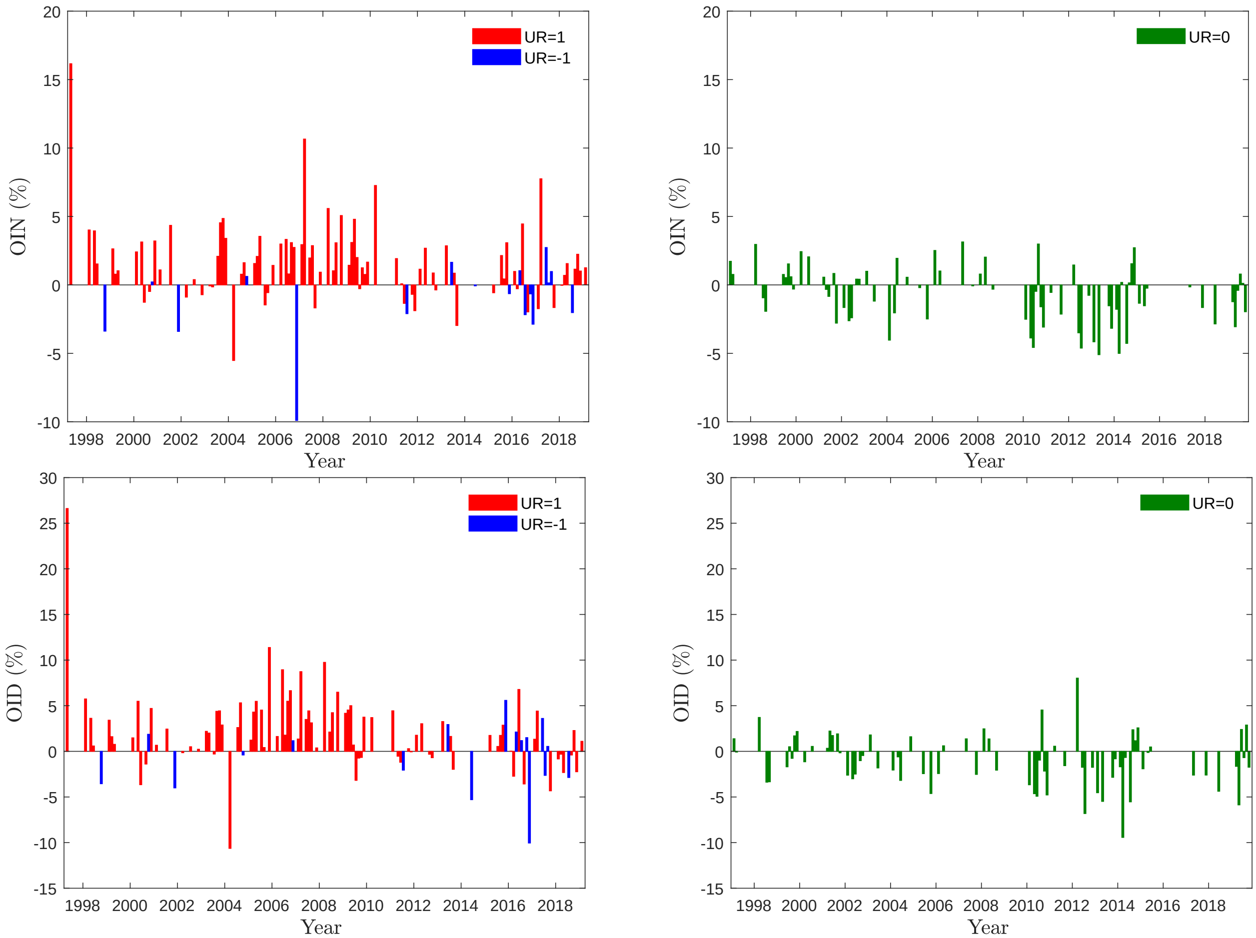
2 compares the average order imbalances in the $[-24H, 0]$ window when uncertainty reduces before FOMC announcements ($UR = \pm 1$) and when uncertainty does not reduce before FOMC announcements ($UR = 0$). The average order imbalances are significantly positive on days with a reduction in pre-FOMC uncertainty, which is consistent with the positive pre-FOMC announcement drift. The difference between the average OIN (OID) of group $UR = \pm 1$ and group $UR = 0$ is 1.99% (2.85%) with a t-stat of 5.11 (5.24) in the 24-hour window before FOMC, which is both statistically and economically significant. The trading activity across uncertainty-reduced and non-uncertainty-reduced announcements shows notable differences, supporting the presence of informed trading before announcements when there is a reduction in uncertainty. The same pattern holds for other pre-event windows, such as $[-12H, 0]$ and $[-24H, -12H]$, which implies that the insider reveals her information gradually.

Next, I assess the statistical significance of these differences. To measure *abnormal* trading activities on announcement days, I also calculate the order imbalances in the same trading hour windows of non-announcement days in the 21 trading days prior to the current FOMC announcement. I regress the two order imbalance measures, OIN and OID on the announcement indicator (ANN) and uncertainty-reduced indicator (UR). Table 3 reports the ordinary least squares (OLS) coefficient estimates.

The UR coefficient estimates in columns 1 (OIN) and 2 (OID) are positive and statistically significant, with t-stats of 5.61 and 4.60, respectively. When there is a reduction in uncertainty on average in the 24-hour window, the number and dollar volume of market orders executed in the direction of the realized pre-FOMC return exceed those in the wrong direction by 1.85% and 2.17% of the total volume, respectively. As shown in columns 3-6, the similar pattern holds for other pre-event windows, such as $[-24H, -12H]$ and $[-12H, 0]$. This finding provides robust evidence of informed trading in the 24-hour window before FOMC announcements when there is a reduction in uncertainty, in line with the private information explanation for the pre-FOMC announcement drift.[10]

[10] Bernile, Hu, and Tang (2016) find evidence consistent with informed trading only until about 30 minutes before scheduled FOMC announcements. The main difference is that we have different definitions of when the insider trading may happen. The insider can profit from the huge uncertainty instead of only upon the large surprise of FOMC news, as in their paper.

Figure 3: Measurement of informed trading: order imbalance



This figure shows the order imbalance based on number of trades (OIN) and dollar volume (OID) of E-mini in the 24-hour window before FOMC announcements. Red (blue) bars represent the order imbalance when uncertainty is reduced, and the cumulative return is positive (negative) over the 24-hour window (i.e., $UR = 1$ ($UR = -1$)). Black bars represent the average order imbalance when there is no reduction in uncertainty in the 24-hour window before FOMC announcements.

3 The benchmark: risk-compensated market makers

Motivated by the above empirical evidence, I introduce the market microstructure with insider trading into a standard macroeconomic framework. The model is a continuous-time version of Kyle's (1985) model with risk-compensated market makers.

3.1 The macroeconomic conditions and information

There are a large number of identical infinitely lived households in the economy. I assume that the aggregate endowment, Y_t, follows

$$\frac{dY_t}{Y_t} = m_t dt + \sigma_Y dB_{Y,t}, \tag{1}$$

where m_t is a continuous-time Ornstein-Uhlenbeck process unobservable to the agent in the economy. The law of motion of m_t is

$$dm_t = a_m \left(\bar{m} - m_t\right) dt + \sigma_{m,t} dB_{m,t}. \tag{2}$$

The standard Brownian motions $B_{Y,t}$ and $B_{m,t}$ in equations (1) and (2), respectively, are independent.

At time 0, the agent's prior belief about m_0 can be represented by a normal distribution. Although m_t is not directly observable, the agent can use two sources of information to update the belief about m_t. First, the realized endowment path contains information about m_t, and second, at pre-scheduled discrete time points $T, 2T, 3T, \cdots$, additional signals about m_t are revealed through announcements. For $n = 1, 2, 3, \cdots$, I denote s_n as the signal observed at time nT and assume $s_n = m_{nT} + \varepsilon_n$, where ε_n is i.i.d. over time and normally distributed with mean zero and variance σ_s^2.

Given the information structure, the posterior distribution of m_t is Gaussian and can be summarized by its first two moments. I define $\hat{m}_t = E_t\left[m_t\right]$ as the posterior mean and $q_t = E_t\left[\left(m_t - \hat{m}_t\right)^2\right]$ as the posterior variance, respectively, of m_t given information up to time t. For $n = 1, 2, \cdots$, in the interior of $\left(nT, \left(n+1\right)T\right)$, the agent updates her belief based on the observed endowment process using the Kalman-Bucy filter:

$$d\hat{m}_t = a_m \left[\bar{m} - \hat{m}_t\right] dt + \frac{q\left(t\right)}{\sigma_Y} d\tilde{B}_{Y,t}, \tag{3}$$

where the innovation process, $\tilde{B}_{Y,t}$ is defined by $d\tilde{B}_{Y,t} = \frac{1}{\sigma_Y}\left[\frac{dY_t}{Y_t} - \hat{m}_t dt\right]$. The posterior variance, $q\left(t\right)$ satisfies the Riccati equation:

$$dq\left(t\right) = \left[\sigma_{m,t}^2 - 2a_m q\left(t\right) - \frac{1}{\sigma_Y^2} q^2\left(t\right)\right] dt. \tag{4}$$

Upon announcements (i.e., at time $t = nT$), the agent updates her belief using Bayes' rule:

$$\hat{m}_{nT}^{+} = q_{nT}^{+}\left[\frac{1}{\sigma_s^2}s_n + \frac{1}{q_{nT}^{-}}\hat{m}_{nT}^{-}\right]; \quad \frac{1}{q_{nT}^{+}} = \frac{1}{\sigma_s^2} + \frac{1}{q_{nT}^{-}}, \tag{5}$$

where $\hat{m}_{nT}^{+}$ and q_{nT}^{+} are the posterior mean and variance after announcements, and $\hat{m}_{nT}^{-}$ and q_{nT}^{-} are the posterior mean and variance before announcements, respectively.

In the standard asset pricing model, FOMC information is revealed only upon announcements ($t = nT$), which results in the realization of the equity premium *at*, *rather than ahead of*, the announcements, as shown in Ai and Bansal (2018). To capture the pre-FOMC announcement drift and informed trading, I introduce a market microstructure with private information into this macroeconomic framework.

3.2 Market microstructure

As in Kyle (1985), the insider in the stock market observes the signal of announcements $s_n = x_{nT} + \varepsilon_n$ at $t = nT - 1$, which happens before FOMC announcements.[11] Thus, she knows the underlying expected growth rate $\hat{m}_{nT}$ and the value of the asset $A(\hat{m}_{nT}, nT)$ earlier than other investors in the market. In addition to the insider, there are noise traders with random, price-inelastic demands. All orders are market orders and are observed by all market makers. Denote by Z_t the cumulative orders of noise traders through time t. The process Z is assumed to be a Brownian motion independent of ε_n, which has mean zero and variance σ_z^2 (per unit of time). Let X_t denote the cumulative orders of the insider, and set $Y = X + Z$.

Given the macroeconomic conditions defined in last section, I assume market makers' announcement-SDF at $t = nT - 1$ is

$$\Lambda_{nT-1,nT}^{*} = \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}_{nT-1}\left[H(\hat{m}_{nT}, nT)\right]}. \tag{6}$$

Discussion of the SDF The SDF can be very general as long as it is countercyclical with respect to the macroeconomic fundamental; that is, $H(\hat{m}_{nT}, nT)$ decreases in the expected growth rate $\hat{m}_{nT}$,

[11] In section 4.1, I relax this assumption such that even the insider is probably informed far ahead instead of only 24 hours before announcements.

which implies that market makers are compensated for the riskiness of assets' fundamentals. The countercyclical SDF is by now standard fare in the literature on macro-finance and can be rationalized by a variety of nonseparable preferences or market frictions (such as heterogeneous beliefs or financial constraints) with standard time-separable preferences. For example, in the appendix, I derive the closed-form announcement SDF under recursive utility where I assume that the aggregate endowment does not instantaneously respond to the FOMC announcements, as in Ai and Bansal (2018). In addition, Ying (2020) shows that under CRRA preferences, heterogeneous beliefs can generate a countercyclical SDF because of the revision in beliefs and the reallocation of consumption when information arrives. Risk-compensated market makers can also be motivated by intermediary asset pricing theory (He and Krishnamurthy (2013, 2018) and Brunnermeier and Sannikov (2014)) with learning, where the information changes the likelihood of the financial constraints being binding and the wealth share of the financial intermediary. Therefore, under a heterogeneous agents framework, the generalized risk sensitivity in Ai and Bansal (2018) is neither a sufficient or necessary condition to generate a positive announcement premium.

The competitive market makers set the price at time $t \in [nT-1,\, nT]$ as

$$P_t = \mathbb{E}\left[\frac{H\left(\hat{m}_{nT}, nT\right)}{\mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right]} A\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right] \tag{7}$$

$$= \frac{\mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right]}{\mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right]} \equiv \frac{V_t}{\Lambda_t}, \tag{8}$$

where I denote by $\mathcal{F}_t^Y$ the information filtration generated by observing the entire past history of aggregate order flow Y and V_t and Λ_t are market makers' estimation of $H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)$ and $H\left(\hat{m}_{nT}, nT\right)$, respectively. Therefore, after observing the cumulative order flow, market makers update the estimation of the asset value as well as the SDF before announcements.

At $t = nT-1$, the market markers have a prior that the expected growth rate upon announcements $\hat{m}_{nT}$ is normally distributed $N\left(\hat{m}_{nT-1}, \Delta Q\right)$ where $\Delta Q = q_{nT-1} - q_{nT}$, as do other agents in the economy (except the insider), as proved in Lemma 5 (see Appendix). Here, $\frac{1}{q_{nT}} = \frac{1}{\sigma_s^2} + \frac{1}{q_{nT-1}}$ from Bayes' rule. I follow the literature to assume that the asset's value of $A\left(\hat{m}_{nT}, nT\right)$ follows a log-normal distribution.[12] More specifically, I specify $\log A(\hat{m}_{nT}, nT) = \beta\hat{m}_{nT} + N(nT)$, where $\beta > 0$ measures how the asset value moves with respect to the fundamental.[13]

Given that the insider knows the expected growth rate $\hat{m}_{nT}$ at $t = nT - 1$, there is no uncertainty surrounding the underlying fundamental since that time. Thus, $\text{A-SDF}^{\text{insider}}_{t,nT} \equiv 1$ under the insider's information set for all $t \in [nT - 1, nT]$. In other words, the insider is risk neutral toward the news contained in announcements thanks to her perfect knowledge of the underlying information.[14] The insider maximizes the expectation of her terminal profit:

$$
\begin{aligned}
& J\left(nT-1, P_{nT-1}, A\left(\hat{m}_{nT}, nT\right)\right) \\
= & \max_{X_t} \mathbb{E}\left[\int_{nT-1}^{nT} \left(A\left(\hat{m}_{nT}, nT\right) - P_t\right) dX_t | \mathcal{F}^Y_{nT-1}, A\left(\hat{m}_{nT}, nT\right)\right] \\
= & \max_{\theta_t \in \mathcal{A}} \mathbb{E}\left[\int_{nT-1}^{nT} \left(A\left(\hat{m}_{nT}, nT\right) - P_t\right) \theta_t dt | \mathcal{F}^Y_{nT-1}, A\left(\hat{m}_{nT}, nT\right)\right].
\end{aligned} \tag{10}
$$

In addition to the entire past history of aggregate order flow Y, the insider knows the actual value of the stock $A(\hat{m}_{nT}, nT)$ and, of course, her own trading. Following Back (1992), I assume that the insider chooses an absolutely continuous trading rule $dX_t = \theta_t dt$ that belongs to an admissible set $\mathcal{A} = \left\{\theta \text{ s.t. } \mathbb{E}\left[\int_{nT-1}^{nT} \theta_s^2 ds\right] < \infty\right\}$. Therefore, the dynamics of aggregate order flow Y are the sum of the insider's demand and the noise traders' demand:

$$dY_t = \theta_t dt + dZ_t.$$

3.3 The equilibrium

Definition 1. *An equilibrium is a price process and an admissible trading strategy, (P_t, θ_t), that satisfy the market makers' rationality condition (7) while solving the insider's optimality condition (10).*

---

[12] It can be extended to a general smooth distribution, as shown in the proof of section C in the Internet Appendix.

[13] One example of the asset's value derived from the standard macroeconomic framework is $A(\hat{m}_t, t) \approx e^{\frac{\phi - 1}{a_m + e^{-\beta}}\hat{m}_t + N(t)}$, where the stock has the claim to the following dividend process:

$$\frac{dD_t}{D_t} = \left[\bar{m} + \phi\left(m_t - \bar{m}\right)\right] dt + \phi\sigma_C dB_{Y,t}. \tag{9}$$

I allow the leverage parameter $\phi \geq 1$ so that dividends are riskier than the endowment, as in Bansal and Yaron (2004).

[14] This can be shown directly through equation (6) where I take the expectation under the insider's information set at $t \in [nT - 1, nT]$.

To solve for an equilibrium, I proceed in a few steps. First, in Lemma 1, conditional on a conjectured insider's trading strategy, I derive the stock price dynamics that are consistent with market makers' filtering. Then, given the assumed dynamics of the equilibrium price, I solve the insider's optimal trading strategy that is captured in Lemma 2 and Lemma 3. Finally, I show that the conjectured rule by market makers is indeed consistent with the insider's optimal choice, as stated in Theorem 1. Without loss of generality, I express $H(\hat{m}_t, t) = e^{-\gamma^A \hat{m}_t + \mathcal{H}(t)}$ with $\gamma_A > 0$ so that the announcement-SDF $\Lambda^*_{nT-1,nT}$ in equation (6) is countercyclical.

Lemma 1. *$\forall t \in [nT-1, nT]$, suppose the insider adopts the following trading strategy:*

$$\theta_t = \frac{\log\left[A\left(\hat{m}_{nT}, nT\right)\right] - \mu_P + \gamma^A \beta \Delta Q}{\left(nT-t\right)\lambda} - \frac{Y_t}{nT-t}, \tag{11}$$

where $\mu_P = \beta \hat{m}_{nT-1} + N(nT)$, $\sigma_v^2 = \beta^2 \Delta Q$, and $\lambda = \frac{\sigma_v}{\sigma_z}$. Then market makers' estimations given by equation (8) satisfy the stochastic differential equations

$$\frac{dV_t}{V_t} = \frac{\beta - \gamma^A}{\beta}\lambda\left[dY_t - \hat{\theta}_t dt\right] \equiv \frac{\beta-\gamma^A}{\beta}\lambda d\hat{Y}_t, \tag{12}$$

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{-\gamma^A}{\beta}\lambda\left[dY_t - \hat{\theta}_t dt\right] \equiv \frac{-\gamma^A}{\beta}\lambda d\hat{Y}_t. \tag{13}$$

The expected insider's order rate under market makers' filtration $\mathcal{F}_t^Y$ is

$$\hat{\theta}_t \equiv \mathbb{E}\left[\theta_t | \mathcal{F}_t^Y\right] = \frac{\gamma^A \beta \Delta Q}{\lambda}, \tag{14}$$

and the adjusted order flow $\hat{Y}_t$,

$$\hat{Y}_t \equiv Y_t - \int_{nT-1}^{t} \hat{\theta}_s \, ds = Y_t - \frac{\gamma^A \beta \Delta Q}{\lambda}\left[t - (nT-1)\right], \tag{15}$$

is a Brownian Motion with instant variance σ_z^2 with respect to market makers' filtration $\mathcal{F}_t^Y$.

Further, market makers' pricing rule in equation (8) is a function of $\left(t, \hat{Y}_t\right)$ that follows

$$\frac{dP\left(t,\hat{Y}_t\right)}{P\left(t,\hat{Y}_t\right)} = \lambda d\hat{Y}_t + \gamma^A \beta \Delta Q dt, \quad \text{with } P_{nT-1} = e^{\mu_P - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2}. \tag{16}$$

When market makers are risk neutral, aggregate order flow Y_t at equilibrium is a martingale under market makers' information set, as shown by Back (1992). In other words, market makers are to set the pricing rule such that the expected order rate from the insider is zero. While market makers are risk averse to the underlying fundamental (i.e., $\gamma^A > 0$), equation (14) indicates that the expected insider's order rate under market makers' filtration $\mathcal{F}_t^Y$ is strictly positive. Here is the intuition behind this result. The information from aggregate order flow resolves market makers' uncertainty before FOMC announcements. Since they are compensated for risk-taking, the equity premium is realized gradually during this period. This leads to an average upward drift in market prices. Therefore, market makers would expect an average positive trading volume from the insider in order to chase that premium. Additionally, the insider has to consider this additional price impact from uncertainty resolution when they trade, which is a unique feature in this model.

The above analysis implies that aggregate order flow Y_t at equilibrium is no longer a martingale under $\mathcal{F}_t^Y$ when $\gamma^A > 0$. More importantly, since the average positive order flow from the insider is expected, market markers would update their estimates from the adjusted order flow $\hat{Y}_t$ instead of aggregate order flow Y_t. Thus, I assume that there exists an equilibrium with two state variables: time t and the adjusted order flow $\hat{Y}_t$. Then, given the market makers' pricing rule, $P(t) = P\left(t, \hat{Y}_t\right)$, the insider chooses the order rate to maximize her trading profit. That is,

$$J\left(t, y, A\left(\hat{m}_{nT}, nT\right)\right) = \max_{\theta_t \in \mathcal{A}} \mathbb{E}\left[\int_t^{nT} \left(A\left(\hat{m}_{nT}, nT\right) - P\left(s, \hat{Y}_s\right)\right) \theta_s ds | \hat{Y}_t = y, A\left(\hat{m}_{nT}, nT\right)\right]$$

subject to

$$d\hat{Y}_t = \left[\theta_t - \hat{\theta}_t\right] dt + dZ_t, \quad \text{where } \hat{\theta}_t \equiv \mathbb{E}\left[\theta_t | \mathcal{F}_t^Y\right]. \tag{17}$$

The principle of optimality implies the following Bellman equation:

$$\max_{\theta_t \in \mathcal{A}} \left\{ \left(A\left(\hat{m}_{nT}, nT\right) - P\left(t, y\right)\right) \theta_t + J_t + J_y \left[\theta_t - \hat{\theta}_t\right] + \frac{1}{2}\sigma_z^2 J_{yy} \right\} = 0, \tag{18}$$

where the subscripts denote the derivatives. The necessary conditions for having an optimal solution to the Bellman equation (18) are

$$J_y\left(t,y,A\left(\hat{m}_{nT},nT\right)\right) = P\left(t,y\right)-A\left(\hat{m}_{nT},nT\right), \tag{19}$$

$$J_t+\frac{1}{2}\sigma_z^2 J_{yy}-\hat{\theta}_t J_y = 0. \tag{20}$$

These necessary conditions lead to the following results.

Lemma 2. *Suppose the expected order rate $\hat{\theta}\left(t\right)=\Theta\left(t,\hat{Y}_t\right)$, where $\hat{Y}_t$ is the adjusted order at t. Let $\omega_t=y$, and suppose that the stochastic differential equation*

$$d\omega_s=dZ_s-\Theta\left(s,\omega_s\right)ds,\quad \forall nT\geq s\geq t\geq nT-1$$

has a unique solution, where Z_s is a Brownian motion with instant variance σ_z^2. If there exists a strictly monotone function $g\left(\cdot\right)$ such that the pricing rule is

$$P\left(t,y\right)=\mathbb{E}\left[g\left(\omega_{nT}\right)\middle|\,\omega_t=y\right], \tag{21}$$

then

$$J\left(t,y,A\left(\hat{m}_{nT},nT\right)\right)=\mathbb{E}\left[j\left(\omega_{nT},A\left(\hat{m}_{nT},nT\right)\right)|\omega_t=y\right] \tag{22}$$

is a smooth solution to Bellman equations (19) and (20), where

$$j\left(y,A\left(\hat{m}_{nT},nT\right)\right)=\int_y^{g^{-1}\left(A\left(\hat{m}_{nT},nT\right)\right)}\left[A\left(\hat{m}_{nT},nT\right)-g\left(x\right)\right]dx\geq 0,\quad \forall\left(y,A\left(\hat{m}_{nT},nT\right)\right).$$

Lemma 3. *Any continuous trading strategy that makes $\lim_{t\to nT}P\left(t,\hat{Y}\left(t\right)\right)=A\left(\hat{m}_{nT},nT\right)$ is optimal, where $P\left(t,y\right)$ is as defined by equation (21).*

Having established these results, I can now proceed to characterize the equilibrium price and the insider's optimal strategy. The equilibrium I obtain, which constitutes the main results of this paper, is summarized in the following theorem.

Theorem 1. *$\forall t\in\left[nT-1,\,nT\right]$, there exists an equilibrium where the price process P_t and optimal strategy* *of the insider θ_t have the dynamics*

$$\frac{dP\left(t,\hat{Y}_t\right)}{P\left(t,\hat{Y}_t\right)} = \lambda d\hat{Y}_t + \gamma^A \beta \Delta Q dt, \tag{23}$$

$$\theta\left(t,\hat{Y}_t\right) = \frac{\log\left[A\left(\hat{m}_{nT}, nT\right)\right] - \mu_P}{\left(nT-t\right)\lambda} - \frac{\hat{Y}_t}{nT-t} + \frac{\gamma^A \beta \Delta Q}{\lambda}, \tag{24}$$

where $\hat{Y}_t$, P_{nT-1}, μ_P, σ_v, and λ are defined in Lemma 1. The expected insider's order rate under $\mathcal{F}_t^Y$ is defined in equation (14).

With respect to the insider's filtration, $P\left(t,\hat{Y}_t\right)$ converges almost surely to $A\left(\hat{m}_{nT}, nT\right)$ at time $t = nT$. When market makers are risk compensated, both the pricing rule $P\left(t,\hat{Y}_t\right)$ and the price-response coefficient $P_{\hat{Y}}\left(t,\hat{Y}\right)$ are submartingales with a constant growth rate $\gamma^A \beta \Delta Q$ under market makers' filtration.

Further, $\forall t \in [nT-1,\, nT]$, the expected cumulative pre-FOMC announcement drift is

$$\log \mathbb{E}\left[\frac{P_t}{P_{nT-1}} | \mathcal{F}_{nT-1}^Y\right] = \gamma^A \beta \Delta Q \left(t - \left(nT-1\right)\right). \tag{25}$$

This implies that there is a strictly positive pre-FOMC announcement drift if and only if market makers are compensated for the riskiness of assets' fundamentals; that is, $\gamma^A > 0$.

I now comment on several implications of the theorem. First, the equilibrium price is a submartingale, expected to increase over time. This contrasts my framework with that in much of the literature. Those studies find the price dynamics are a martingale under risk-neutral market makers since they are indifferent toward resolving uncertainty either now or in the future. When market makers are risk compensated, however, the resolution of uncertainty is associated with the realizations of the premium. The positive expected pre-FOMC announcement premium is cumulated at a constant rate $\gamma^A \beta \Delta Q$, which is the negative covariance between the innovation to the A-SDF and the asset value. Intuitively, the pre-announcement drift would be larger: (1) when market makers are more risk averse to the underlying fundamental, (2) when the asset value has a greater exposure to FOMC news, and (3) when FOMC announcements are more transparent, which reduces uncertainty even more. In addition, the equilibrium price converges to the value $A\left(\hat{m}_{nT}, nT\right)$, known ex ante only to the insider, at FOMC announcements. This guarantees that all of the private information will eventually be incorporated into the price and generalizes the result proved in Back (1992).

Second, I find that when market makers are risk averse to the underlying fundamental (i.e., $\gamma^A > 0$), the expected insider's order rate under market makers' filtration $\mathcal{F}_t^Y$ follows

$$\mathbb{E}\left[\theta_t|\mathcal{F}_t^Y\right] = \frac{\gamma^A \beta \Delta Q}{\lambda} = \gamma^A \sqrt{\Delta Q}\sigma_z, \tag{26}$$

which is strictly positive. This is different from Kyle-type models, where the expected insider's order rate is always zero. Here is the intuition. Because of the average upward drift in market prices, market makers rationally anticipate that the insider would trade positively on average to chase that premium. The insider also has to consider the additional price impact from an uncertainty resolution when she trades, which is unique in this model. The equilibrium expected insider's order rate is determined by the ratio of the expected pre-FOMC announcement premium per unit of time ($\gamma^A \beta \Delta Q$) to Kyle's lambda (λ). Therefore, the abnormal order imbalances are positive on average when there is private information before FOMC announcements. In addition, the insider would, on average, trade more aggressively when market makers are more risk averse to uncertainty or FOMC announcements are more transparent, which is associated with the higher realized equity premium per unit of time. In the meantime, when noise traders are more active, the insider trades more on average because of the smaller price impact, which has been largely missed in Kyle-type models.[15]

Third, the price impact $P_{\hat{Y}}\left(t, \hat{Y}\right)$ is also a submartingale, which grows at the same rate as the equilibrium price. Risk-averse market makers benefit from an uncertainty resolution out of the observation of aggregate order flow. Therefore, to entice the insider to trade and release information early, market makers have incentives to set a price impact that increases on average. Collin-Dufresne and Fos (2016) is one of the few papers that achieve the similar result through a different channel that comes from the insider's potential benefit from waiting for better liquidity with stochastic noise trading volatility.

Fourth, when market makers converge to be risk neutral, the limit of the equilibrium is well defined and converges to the traditional Kyle model (more precisely, it converges to Back (1992)).

[15] The only exception is the model of Collin-Dufresne and Fos (2016), which derives the similar result except they rely on the assumption that noise trading volatility follows a stochastic process.

When γ^A converges to zero, the expected insider's order rate $\hat{\theta}_t$ converges to zero, which implies that aggregate order flow Y_t converges to a martingale. The pricing rule $P\left(t, \hat{Y}_t\right)$ and the price-response coefficient $P_{\hat{Y}}\left(t, \hat{Y}\right)$ also converge to martingales. This implies that the expected pre-announcement drift converges to a flat line, as in Back (1992). The convergence result demonstrates that there is a strictly positive pre-FOMC announcement drift if and only if market makers are compensated for the riskiness of assets' fundamentals (i.e., $\gamma^A > 0$), as proved in Theorem 1.

3.4 Properties of equilibrium

Having characterized the equilibrium, in this section, I study the equilibrium properties and map the model to asset market fluctuations before FOMC announcements.

The following proposition captures an uncertainty reduction prior to announcements in the equilibrium from Theorem 1.

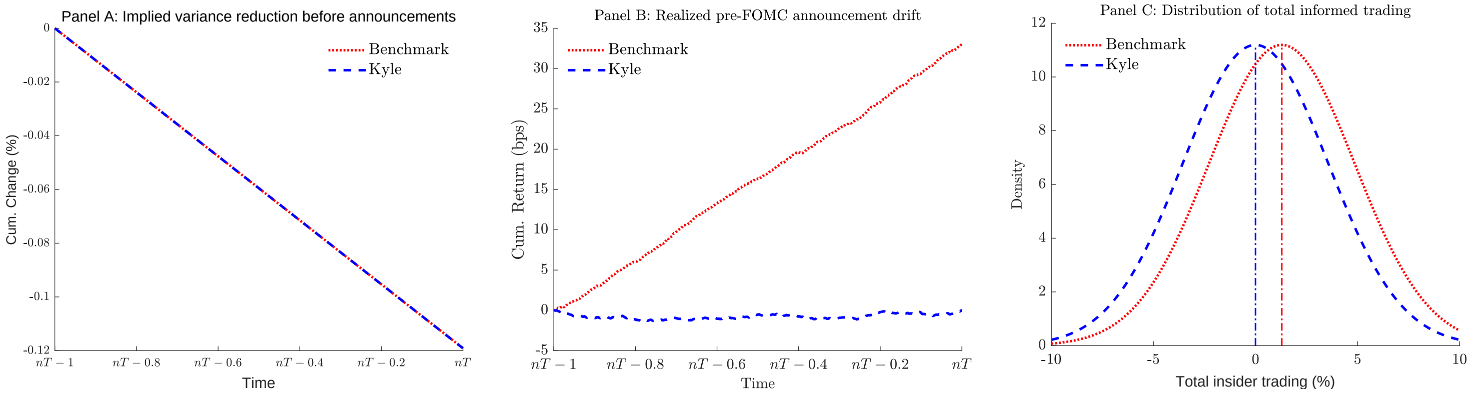
Proposition 1. *With respect to market makers' filtration $\mathcal{F}_t^Y$, $\forall t \in [nT-1, nT]$, an uncertainty reduction at time t compared to $nT-1$, follows*

$$Var\left[\log P_{nT} | \mathcal{F}_t^Y\right] - Var\left[\log P_{nT} | \mathcal{F}_{nT-1}^Y\right] = -\beta^2 \Delta Q\left[t - (nT-1)\right]. \tag{27}$$

Thus, prior to announcements, uncertainty is reduced at a constant rate $\beta^2 \Delta Q$ per unit of time.

Figure 4 plots the model implications. I refer to the case where market makers are risk compensated as the benchmark. For comparison, I study another case where I keep other parameters the same and assume market makers are risk-neutral, which is equivalent to the original Kyle model with a log-normal distribution of the asset value (see Back (1992)). Panel A plots the implied variance changes before announcements that are the same for both cases. This is because the implicated variance reduction only depends on the risk exposure β, which is not a function of risk aversion γ^A. However, the average realized pre-FOMC announcement excess returns are different, as shown in Panel B. There is a strictly positive pre-FOMC announcement drift if and only if market makers are compensated for the riskiness of assets' fundamentals, in line with the proof.

Figure 4: Model implications: uncertainty, return, and informed trading



This figure shows the model implications for the case with risk-compensated market makers (Benchmark) and with risk-neutral market makers (Kyle) as a function of time, respectively. Panels A and B plot the average implied variance change and average realized pre-FOMC announcement return before announcements, which are computed from 10,000 parallel samples. Panel C shows the distribution of the total informed trading $\int_{nT-1}^{nT} \theta_t \, dt$ of each announcement for both cases. The parameters are as follows: β=3, γ_A=112, σ_z=13.35%, σ_Y=3.16%, σ_s=0.4%, $\sigma_{m,t} \equiv \sigma_{m,0}$=0.3%, $\bar{m}$=1.5%, a_m=4.5%.

To explain informed trading, as documented in Figure 3 and Table 2, I plot the distribution of the total informed trading $\int_{nT-1}^{nT} \theta_t \, dt$ of each announcement for both cases in Panel C of Figure 4.[16] From Lemma 1, it is straightforward to show that $\int_{nT-1}^{nT} \theta_t \, dt$ is normally distributed $N(\frac{\gamma^A \beta \Delta Q}{\lambda}, 2\sigma_z^2)$ in the benchmark and $N(0, 2\sigma_z^2)$ in the Kyle model. In other words, the average informed trading in the benchmark $\frac{\gamma^A \beta \Delta Q}{\lambda}$ is significantly positive, as in the data, whereas it is zero in the Kyle model. The significant positive informed trading comes from the positive excess return before FOMC announcements. Therefore, risk-compensated market makers are also necessary and sufficient to account for the positive informed trading before announcements.

4 Extensions

To explain the timing of the pre-FOMC announcement drift, in subsection 4.1, I extend the benchmark and allow the insider to choose the starting time such that the informed profits are the highest. Subsection 4.2 extends the benchmark such that the insider may or may not have private information and market makers do not know it, which explains the time-series pre-FOMC announcement drift

[16] I can also define informed trading as order imbalances in the data: $OI_B = \int_{nT-1}^{nT} p_t \theta_t 1_{\{p_t > p_{t-1}\}} \, dt$ and $OI_S = \int_{nT-1}^{nT} p_t \theta_t 1_{\{p_t < p_{t-1}\}} \, dt$, which generates similar implications.

and uncertainty reduction.

4.1 The timing of the pre-FOMC announcement drift

To fully account for the pre-FOMC announcement drift, timing is another puzzle that needs to be explained: Why does it occur 24 hours prior to announcements when private information is probably known way before?

In this subsection, I extend the benchmark model such that the insider knows private information earlier than $nT-1$ and chooses the starting time $s \in [(n-1)T, nT)$ to maximize her unconditional expected profits, where $(n-1)T$ is the time just after the last announcement, and nT is the time that the following announcement will be made. Once she starts to trade, she will keep trading, as in Kyle-type models. The equilibrium is captured in the following theorem.

Theorem 2. *When the insider starts to trade at $s \in [(n-1)T,\, nT)$, the equilibrium price process P_t and optimal strategy of the insider θ_t follow*

$$\frac{dP\left(t,\hat{Y}_t\right)}{P\left(t,\hat{Y}_t\right)} = \lambda_s d\hat{Y}_t + \frac{\gamma^A \beta \Delta Q_s}{nT-s}dt, \tag{28}$$

$$\theta\left(t,\hat{Y}_t\right) = \frac{\log\left[A\left(\hat{m}_{nT}, nT\right)\right]-\mu_P}{\left(nT-t\right)\lambda_s} - \frac{\hat{Y}_t}{nT-t} + \frac{\gamma^A \beta \Delta Q_s}{(nT-s)\lambda_s}. \tag{29}$$

Here $\Delta Q_s = q_s - q_{nT}$, $\Sigma_s = \beta^2 \Delta Q_s$, $\sigma^2_{v,s} = \frac{\Sigma_s}{nT-s}$, $\lambda_s = \frac{\sigma_{v,s}}{\sigma_z}$. The initial price at time s is $P_s = e^{\mu_{P,s}-\frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\Sigma_s+\frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\Sigma_s}$ where $\mu_{P,s} = \beta\hat{m}_s + N\left(nT\right)$.

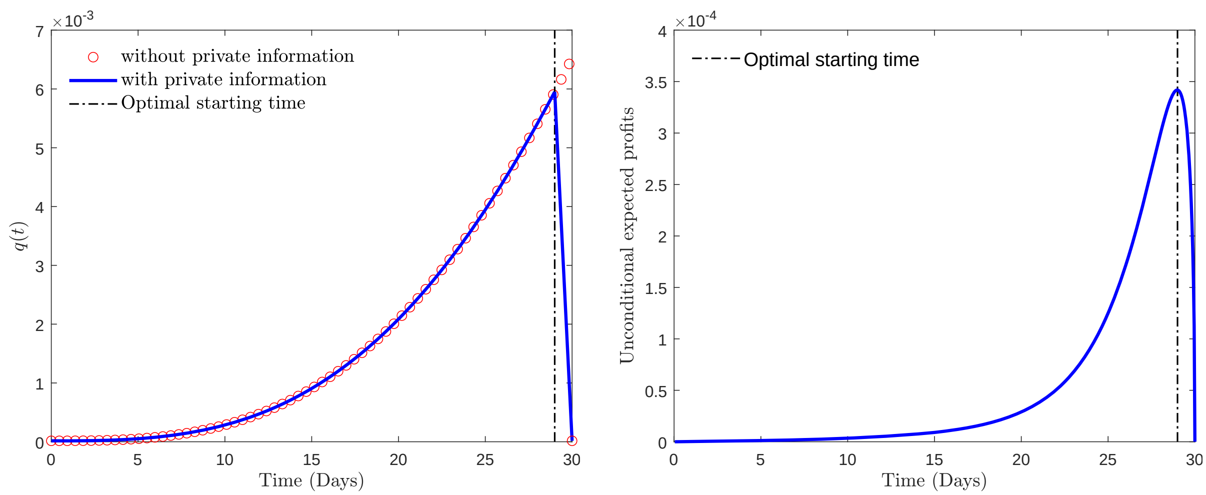
The unconditional expected profits of the insider that starts to trade at $s \in [(n-1)T,\, nT)$ are

$$\mathbb{E}\left[J\left(s, P_s, A\left(\hat{m}_{nT}, nT\right)\right)\right] = \frac{e^{\mu_{P,s}+\frac{1}{2}\Sigma_s}}{\lambda_s}\left(\frac{\beta+\gamma^A}{\beta}\Sigma_s - 1 + e^{-\frac{\gamma^A}{\beta}\Sigma_s}\right), \tag{30}$$

where $\mathbb{E}\left[J\left((n-1)T, P_{(n-1)T}, A\left(\hat{m}_{nT}, nT\right)\right)\right] = \mathbb{E}\left[J\left(nT^-, P_{nT^-}, A\left(\hat{m}_{nT}, nT\right)\right)\right] = 0.$

Theorem 2 indicates that the unconditional expected profits follow an inverted U shape as they rise and then fall again when the next announcement approaches. This is because the insider faces a trade-off between uncertainty and liquidity when she decides when to trade. The insider's expected profit from the asymmetric information increases with market uncertainty because the insider has relatively more private information when the market is noisier. When $s = (n-1)T$, the insider does not have any information advantage since $\Sigma_{(n-1)T} = 0$. Uncertainty increases in time before the next announcement since more noisy signals $B_{m,t}$ are released from the dynamics of the growth rate in equation (2), where the law of motion of posterior variance $q(t)$ is captured in equation (4). This is consistent with Figure 1 that market uncertainty increases before FOMC announcements and does not decrease until the insider starts to trade. Therefore, upon receiving private information, the insider would like to trade later in a noisier market instead of trading immediately. However, she cannot trade too late since she needs substantial liquidity trading to hide her information. Otherwise, the price impact $\lambda_s = \frac{\sigma_{v,s}}{\sigma_z} = \frac{\sqrt{\frac{\Sigma_s}{nT-s}}}{\sigma_z}$ goes to infinity as $s \to nT^-$, and the expected profit converges to zero.

Figure 5: The timing of the pre-FOMC announcement drift



This figure plots the posterior variance of the market maker's belief of growth rate m_t and the unconditional expected profits when the insider starts to trade at a given time belong to $[(n-1)T, nT)$. The announcements happen every 30 days. The black vertical line indicates the starting time that the informed profits are highest, which is 24 hours before FOMC announcements. In the left panel, the red circles capture the dynamics of the posterior variance when all the information is revealed upon FOMC announcements. The blue line captures the dynamics of posterior variance when she optimally chooses the starting time to trade. Here, I assume $\sigma_{m,t} = \sigma_{m,(n-1)T} + \kappa_m(t-(n-1)T)$ for $t \in [(n-1)T, nT)$ with $\kappa_m = 6$. All other parameters are the same as in the benchmark, as reported in Table 4.

Figure 5 shows the dynamics of the posterior variance of the growth rate m_t from the market maker's perspective and the unconditional expected profits of the insider when she chooses when to start to trade. To quantitatively capture the notion that the posterior variance increases faster in time, as shown in Figure 1, I assume $\sigma_{m,t} = \sigma_{m,(n-1)T} + \kappa_m(t - (n-1)T)$ where $\kappa_m = 6$ and all other parameters are the same as in the benchmark. When there is no private information before FOMC announcements, the posterior variance q_t follows equation (4), captured by the red circles in the left panel of Figure 5. The right panel plots the unconditional expected profits when the insider chooses to trade at any time between $(n-1)T$ and nT. When I vary the starting time of informed trading in my calibration, I find that informed profits are highest when the starting time is 24 hours before the announcement. The blue line in the left panel shows the dynamics of posterior variance q_t under the optimal starting time. Therefore, my paper also explains the timing of the pre-FOMC announcement drift, which is another important feature of the pre-FOMC puzzle.

4.2 Uncertain informed trading

Figure 2 indicates that not all FOMC announcements are the same—some are not associated with an uncertainty reduction prior to announcements. Motivated by this fact, I extend the benchmark such that the insider may or may not be informed of the signal s_n before announcements.[17] In the meantime, market makers are not sure whether or not the insider observes the signal. Market makers share a common belief that such an event, in which the insider observes this information earlier than the public, occurs with probability $\pi_{nT-1} \in (0,1)$ at time 0. Therefore, in addition to the discounted value of the risky asset and the A-SDF, market makers also have to update their estimate of the probability that the insider has private information about FOMC announcements.

4.2.1 Model setting

Let $X_{\delta,t}$ denote the net orders from the insider. The total cumulative order flow Y_t is expressed as

$$Y_t = X_{\delta,t} + Z_t,$$

where δ is an indicator function, which is equal to 1 if the insider has information and is equal to 0 otherwise. By observing this order flow, market makers update their estimates about the *probability*

[17] This extension is based on Li (2013), which assumes risk-neutral market makers.

that the insider possesses private information and the *value* of the risky security. Let $\mathcal{F}_{0,t} = \mathcal{F}_t^Y \times \{\delta = 0\}$ under the hypothesis $\delta = 0$ and $\mathcal{F}_{1,t} = \mathcal{F}_t^Y \times \{\delta = 1\}$ under the hypothesis $\delta = 1$. I let $\pi(t) = \mathbb{E}\left[\delta | \mathcal{F}_t^Y\right]$ be the estimate of the probability that the insider has private information at time t.

If the insider does not have any private information ($\delta = 0$), she has no information other than what the market makers have. Therefore, $\forall t \in [nT-1, nT]$, the best estimate of the security's value is

$$\begin{aligned}\bar{v}^* &\equiv \mathbb{E}\left[\frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}\right]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}\right], \\ &= \frac{\mathbb{E}\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}\right]}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}\right]} \equiv \frac{\bar{V}}{\bar{\Lambda}},\end{aligned} \tag{31}$$

where I define $\bar{V}$ and $\bar{\Lambda}$ as the estimate of $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$ and $A(\hat{m}_{nT}, nT)$ under the case that the insider is not informed, respectively.

If the insider has private information ($\delta = 1$), the value estimate of the risky security at time t conditional on $\delta = 1$ is

$$\begin{aligned}v^*(t) &\equiv \mathbb{E}\left[\frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}\right]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}\right], \\ &= \frac{\mathbb{E}\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}\right]}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}\right]} \equiv \frac{V(t)}{\Lambda(t)},\end{aligned} \tag{32}$$

where I define $V(t)$ and $\Lambda(t)$ as the estimate of $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$ and $A(\hat{m}_{nT}, nT)$ under the case that the insider is informed, respectively.

With uncertainty of δ, market makers estimate the discounted value under the information structure $\mathcal{F}_{1,t}$ and estimate the probability that the insider has observed private information under the information structure $\mathcal{F}_t^Y$. Given these two estimates, market makers set the price that follows

$$\begin{aligned} P(t) &= \mathbb{E}\left[\frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) \mid \mathcal{F}_t^Y\right]} A(\hat{m}_{nT}, nT) \mid \mathcal{F}_t^Y\right], \\ &= \frac{\mathbb{E}\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) \mid \mathcal{F}_t^Y\right]}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) \mid \mathcal{F}_t^Y\right]}, \\ &= \frac{\pi(t) V(t) + (1-\pi(t)) \bar{V}}{\pi(t) \Lambda(t) + (1-\pi(t)) \bar{\Lambda}}. \end{aligned} \tag{33}$$

Note that when market makers know the insider is always informed ($\pi_{nT-1} = 1$), market makers set the price as

$$P(t) = \mathbb{E}\left[\frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}\left[H(\hat{m}_{nT}, nT) \mid \mathcal{F}_{1,t}\right]} A(\hat{m}_{nT}, nT) \mid \mathcal{F}_{1,t}\right],$$

which goes back to the benchmark model in section 3.

I impose the following restriction on market makers' value estimates $V(t)$ and $\Lambda(t)$ conditional on $\delta = 1$ defined in equation (32):

$$\mathbb{E}\left[\int_{nT-1}^{nT} V^2(s)\, ds\right] < \infty; \quad \mathbb{E}\left[\int_{nT-1}^{nT} \Lambda^2(s)\, ds\right] < \infty.$$

This restriction implies that the pricing rule defined by equation (33) satisfies

$$\mathbb{E}\left[\int_{nT-1}^{nT} P^2(s)\, ds\right] < \infty,$$

which is sufficient to rule out the so-called doubling strategy that the insider could use.

4.2.2 The equilibrium

Definition 2. *An equilibrium is an quadruple (X_0, X_1, P, Π) such that*

1. *both X_0 and X_1 are the optimal trading strategies of the insider when she has not or has observed private information, respectively, given $P(t)$ and Π;*

2. *$P(t) = \frac{\Pi(t)V(t)+(1-\Pi(t))\bar{V}}{\Pi(t)\Lambda(t)+(1-\Pi(t))\bar{\Lambda}}$ is the stock price at time t, where $V(t)$ and $\Lambda(t)$ are market makers' value estimates of the risky security and SDF conditional on $\delta = 1$, and $\Pi(t) = \pi(t)$ is market makers'*

probability estimates that the insider has private information, given the insider's trading strategies X_0 and X_1.

As in the benchmark in section 3, I assume that the insider chooses an absolutely continuous trading rule,

$$dX_{1,t} = \theta\left(t, \tilde{V}\right) dt,$$

where $\theta\left(t, \tilde{V}\right)$ belongs to an admissible set $\mathcal{A} = \left\{\theta \text{ s.t. } \mathbb{E}\left[\int_{nT-1}^{nT} \theta^2\left(t, \tilde{V}\right) ds\right] < \infty\right\}$ and $\tilde{V}$ is the insider's perfect knowledge of $H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)$. When the insider has no information other than what the market makers have, her order rate becomes $\theta\left(t, \bar{V}\right)$. Given this trading strategy, the cumulative flow is

$$Y_t = \int_{nT-1}^{t} \theta\left(s, V\right) ds + Z_t. \tag{34}$$

From market makers' point of view, the cumulative order flow has two possible interpretations because they don't know how much noise traders trade. One is

$$dY_t = \theta\left(t, \tilde{V}\right) dt + dZ_t, \tag{35}$$

if the insider is informed, and the other is

$$dY_t = \theta\left(t, \bar{V}\right) dt + dZ_t, \tag{36}$$

if the insider is not informed.

The following assumption imposes that when the insider is not informed, she will not take a dramatically different trading strategy. Otherwise, her trading behavior may immediately reveal that she does not have private information for a specific FOMC announcement.[18]

Assumption 1. *When the insider is not bettered informed, she maximizes the following terminal profit under*

[18] It is possible that the insider observes a private signal, indicating that the terminal value is $\bar{v}^*$. Under this case, since the insider has no information advantage compared to market makers' prior, I interpret it as the insider having no private information.

her best estimation of the asset value:

$$\int_{nT-1}^{nT}\left(\mathbb{E}\left[\frac{H\left(\hat{m}_{nT},nT\right)}{\mathbb{E}\left[H\left(\hat{m}_{nT},nT|\mathcal{F}_{nT-1}\right)\right]}A\left(\hat{m}_{nT},nT\right)|\mathcal{F}_{nT-1}\right]-P_s\right)\theta_s ds$$
$$=\int_{nT-1}^{nT}\left(\bar{v}^*-P_s\right)\theta_s ds.$$

Given the observation of the cumulative order flow, market makers update the probability that the insider has private information, the A-SDF, as well as the discounted value of the security conditional on the insider being informed. These estimates are done by solving a nonlinear filtering problem. The equilibrium is summarized in the following theorem (proved in the Appendix).

Theorem 3. *Under Assumption 1, $\forall t \in [nT-1,\ nT]$, there exists an equilibrium (X_0, X_1, P, Π) as follows:*

(1). Market makers' probability estimate $\Pi(t,y)$ is

$$\Pi(t,y)=\frac{\pi_{nT-1}\exp\left(\frac{1}{2\sigma_z^2}\frac{[y-\bar{y}]^2}{nT-t}+\frac{1}{2}\log(nT-t)-\frac{\bar{y}^2}{2\sigma_z^2}\right)}{1-\pi_{nT-1}+\pi_{nT-1}\exp\left(\frac{1}{2\sigma_z^2}\frac{(y-\bar{y})^2}{nT-t}+\frac{1}{2}\log(nT-t)-\frac{\bar{y}^2}{2\sigma_z^2}\right)},\tag{37}$$

where y represents the adjusted order flow $\hat{Y}_{1,t}$ (defined later) and $\bar{y}=\frac{\frac{\beta-\gamma^A}{\beta}\sigma_v^2}{2\lambda}$;

(2). The pricing rule $P(t,y)$ has dynamics

$$P(t,y)=P_{nT-1}\frac{\Pi(t,y)\,e^{\frac{\beta-\gamma^A}{\beta}\lambda y-\frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2(t-(nT-1))}+1-\Pi(t,y)}{\Pi(t,y)\,e^{-\frac{\gamma^A}{\beta}\lambda y-\frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2(t-(nT-1))}+1-\Pi(t,y)},\tag{38}$$

where P_{nT-1}, σ_v, and λ are defined in Lemma 1;

(3). The insider's trading strategy $X_\delta(t,y)$ satisfies

$$X_1(t,y)=\int_{nT-1}^{t}\theta\left(s,y;H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)\right)ds,\quad X_0(t,y)=\int_{nT-1}^{t}\theta\left(s,y;\bar{V}\right)ds.\tag{39}$$

The insider's order rate for any $\tilde{V}$ is

$$\theta\left(t,y;\tilde{V}\right)=\bar{\theta}(t,y)+\frac{\left(\log\tilde{V}-\mu_V\right)/\left(\frac{\beta-\gamma^A}{\beta}\lambda\right)-\bar{y}-\Pi(t,y)\left[y-\bar{y}\right]}{nT-t},$$

where $\mu_V = \left(\beta - \gamma^A\right)\hat{m}_{nT-1} + \mathcal{H}(nT) + N(nT)$ *is the mean of* $\log\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)\right]$.

The expected order rate of the insider $\bar{\theta}(t,y)$ *under market makers' filtration* $\mathcal{F}_t^Y$ *satisfies*

$$\bar{\theta}(t,y) \equiv \mathbb{E}\left[\theta\left(t,y;\tilde{V}\right)|\mathcal{F}_t^Y\right] = \frac{\frac{\gamma^A\beta\Delta Q}{\lambda}\Pi(t,y)E(t,y) - \Pi(t,y)\left(1-\Pi(t,y)\right)\frac{y-\bar{y}}{nT-t}\left(E(t,y)-1\right)}{\Pi(t,y)\cdot E(t,y) + 1 - \Pi(t,y)}, \tag{40}$$

where $E(t,y) = e^{-\frac{\gamma^A}{\beta}\lambda y - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2(t-(nT-1))}$.

The adjusted order flow $\hat{Y}_{1,t}$ *starts from 0 and follows*

$$d\hat{Y}_{1,t} = \frac{\left(\log\tilde{V} - \mu_V\right)/\left(\frac{\beta-\gamma^A}{\beta}\lambda\right) - \hat{Y}_{1,t}}{nT-t}dt + dZ_t, \tag{41}$$

where $\tilde{V} = H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)$ *when informed and* $\tilde{V} = \bar{V}$ *when not informed.*

Theorem 3 shows that the benchmark's main results still hold when I extend the model with the potential better-informed insider. Conditional on the insider being better informed, the equilibrium price dynamics are a submartingale when market makers are risk-compensated. Both the growth rate of the expected pre-FOMC announcement drift and the expected insider's order rate are time varying because of the dynamics of the probability estimate. In addition, the pricing rule is nonlinear and stochastic, which drives the price volatility, market depth, and price response to be stochastic.

4.2.3 Properties of equilibrium and model calibration

In this section, I study the equilibrium properties and calibrate the model to the time series pattern of the pre-FOMC announcement drift.

Proposition 2. *For any smooth distribution of the prior* $G\left(\pi_{nT-1}\right)$, *the average realized pre-FOMC announcement drift just before announcements* $\left(t = nT^-\right)$ *is*

$$\log\mathbb{E}\left[\frac{P_{nT^-}}{P_{nT-1}}\right] = \eta\gamma^A\beta\Delta Q,$$

where the expectation is taken over all states of natural and η *is the fraction of the announcements that the* *insider is informed. Meanwhile, the average uncertainty reduction just before announcements is*

$$\mathbb{E}\left[Var\left[\log P_{nT^-}|\mathcal{F}^Y_{nT^-}\right]-Var\left[\log P_{nT^-}|\mathcal{F}^Y_{nT-1}\right]\right]=-\eta\beta^2\Delta Q.$$

Here, the expectation is also taken over all states of natural.

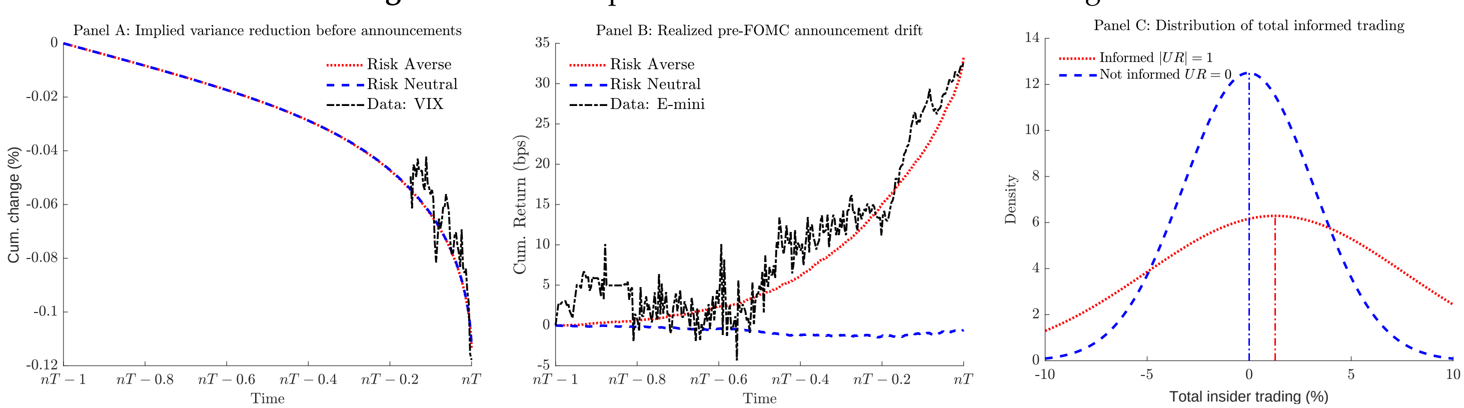
Proposition 2 captures the average realized pre-FOMC announcement drift and the average uncertainty reduction just before announcements in the presence of the potential better-informed insider. The intuition is as follows. When the insider is not informed, market makers figure that out just before the announcement (i.e., $\lim_{t\to nT^-}\Pi\left(t,\hat{Y}_{1,t}\right)=0$) and the price P_{nT^-} equals P_{nT-1} almost surely. Moreover, there is no uncertainty reduction since the insider has no information other than what market makers have at $t=nT-1$. When the insider is better informed, however, all of the private information is eventually incorporated into the price, which is associated with an uncertainty reduction. The probability estimate converges to 1 and the price converges to $A\left(\hat{m}_{nT},nT\right)$ almost surely upon announcements.

Calibration The closed-form solutions in Theorem 3 and Proposition 2 generate a precise mapping from the model's parameters to the asset market evidence before FOMC announcements. The calibration is summarized in Table 4. I begin by calibrating external parameters by setting the long run output growth rate to $\bar{m}=1.50\%$, the volatility of aggregate output to $\sigma_Y=3.16\%$, and the persistence and volatility of the growth rate to $a_m=4.5\%$ and $\sigma_m=0.40\%$, all common values in the literature. The preference parameters are taken from the standard long-run risk literature. I impose the transparency of announcements $\sigma_s=0.45\%$ to match the total uncertainty reduction before FOMC. Since 58% of the 187 announcements from 1996 to 2019 are associated with pre-FOMC uncertainty reduction, I choose the fraction of FOMC announcements that the insider is informed η to be 58%. I choose the exposure of the risky asset $\beta=3$ to match the level of the pre-FOMC announcement drift and the volatility of noise traders $\sigma_z=17.9\%$ to match the average total informed trading conditional on an uncertainty reduction before FOMC announcements. I assume market makers' prior $\pi_{nT-1}\equiv 20\%$ to match the nonlinear trend of the pre-FOMC announcement drift.

Panels A and B of Figure 6 plot the average uncertainty reduction and the average realized pre-

FOMC announcement drift 24 hours before announcements in the model and the data. To examine the overnight price dynamics, I calculate the pre-FOMC announcement drift by E-mini instead of the S&P 500 index. VIX is only allowed to trade during regular trading hours between 9:30 a.m. and 4:15 p.m. ET before April 2016. There are no overnight data. Thus, I plot the implied variance reduction from 3 hours before FOMC announcements compared to that 24 hours before announcements. The dotted red lines indicate the case under calibrated parameters. For comparison, I also show the case under risk-neutral market makers (the dashed blue lines) and keep other parameters the same. Both cases match the uncertainty reduction well. However, only under risk-averse market makers, is there a strictly positive pre-FOMC announcement drift that is consistent with the data. Additionally, the dotted red line matches the time series pattern of the pre-FOMC announcement drift fairly well such that the cumulative return grows faster when approaching the time of FOMC announcements. The intuition is as follows. When the insider is informed, as time goes by, the cumulative order flow reveals more private information that speeds up the probability estimation of market makers. This results in a faster uncertainty reduction, which is associated with the deeper pre-FOMC announcement drift.

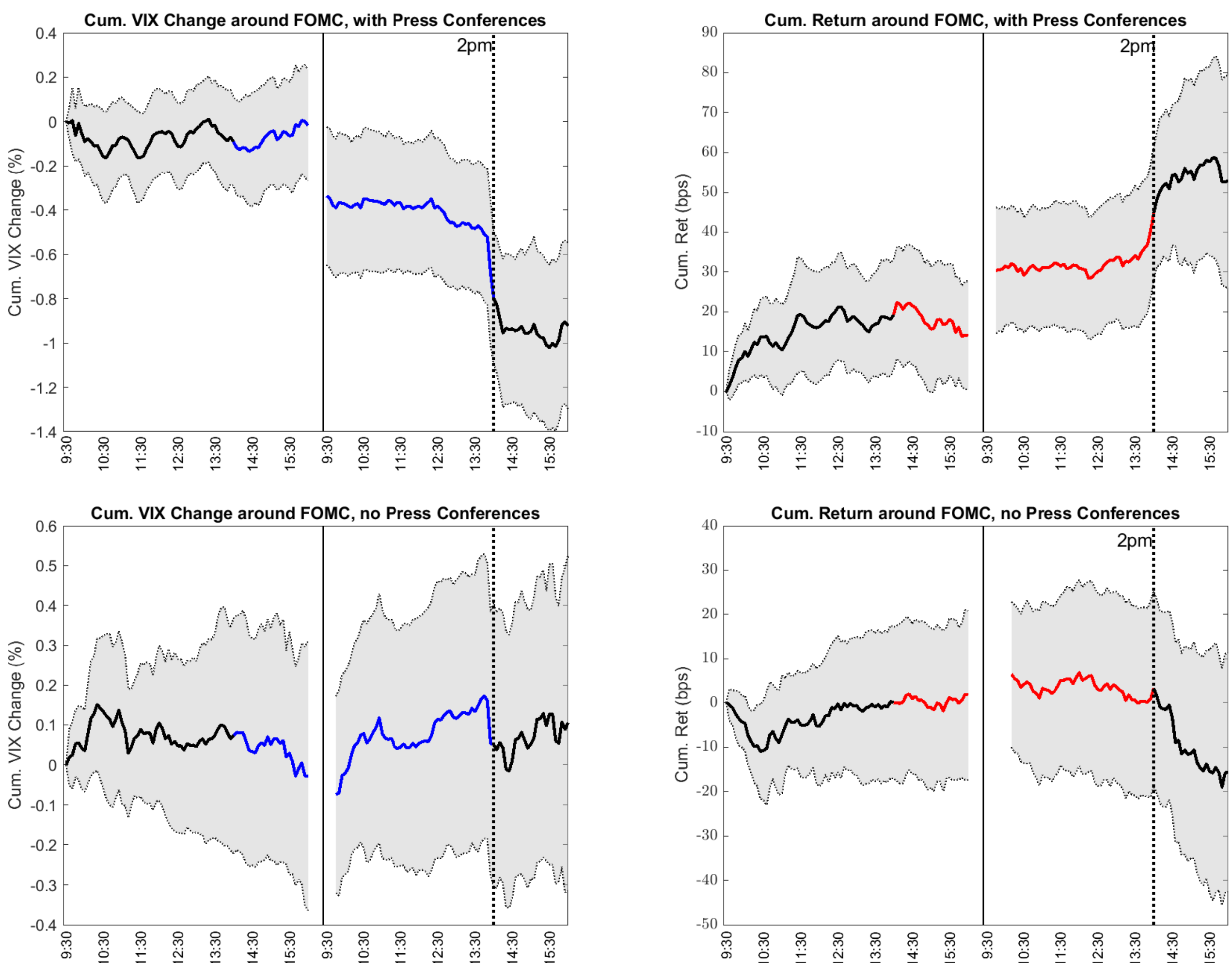
Figure 6: Model implications: uncertain informed trading



The figure plots the model implications where the insider is potentially better informed, which are computed from 10,000 parallel samples. In Panels A and B, the dotted red lines and the dashed blue lines indicate the cases with risk-averse market makers and risk-neutral market makers, respectively. The black lines show the change of VIX^2 and the cumulative return of E-mini around FOMC announcements in the data. Panel C shows the distribution of the total informed trading $\int_{nT-1}^{nT} \theta_t \, dt$ of each announcement with risk-averse market makers, conditional on whether or not the insider is informed. The parameters are reported in Table 4.

Panel C of Figure 6 plots the distribution of the total informed trading $\int_{nT-1}^{nT} \theta_t \, dt$ in the full model. When the insider has private information, the mean of informed trading is 1.27%, as that in the data, which is strictly positive since the insider trades to chase the pre-FOMC announcement drift. When the insider is not informed, the average is zero and less volatile since her estimation of the asset's value is always $\bar{v}^*$. The model's implications are consistent with the empirical facts shown in Figure 3.

Figure 7: Classifications of FOMC meetings: press conferences



This figure shows the average cumulative return in the S&P 500 index in two-day windows with and without press conferences from April 2011 to December 2019. The blue (red) solid line indicates the VIX change (cumulative return of the SPX) on the 2 p.m. to 2 p.m. pre-FOMC window.

5 Further implications

In addition to the empirical facts shown in section 2, in this section, I demonstrate that other asset market fluctuations around FOMC announcements are consistent with the model's predictions.

5.1 Two distinctive patterns to equity returns: press conferences

Since April 2011, the chair of the FOMC has been holding a press conference at every other FOMC meeting.[19] At these meetings the FOMC also releases a document called Summary of Economic Projections (SEP) from its members. Three forms of communication take place: the FOMC statement, the SEP, and the press conference with the chair.

Boguth, Gregoire, and Martineau (2019) study the impact of the press conferences and find that the pre-FOMC announcement drift is limited to announcements with press conferences since April 2011, as shown in the right panels of Figure 7. To explain the two distinctive patterns, in the top left panel, I document that uncertainty (measured by VIX) decreases significantly before announcements with press conferences. When no press conferences take place, however, uncertainty on average does not decrease before FOMC announcements. This finding is consistent with the full model that only when uncertainty decreases before announcements from informed trading, is there a positive pre-FOMC announcement drift. In addition, intuitively, when the upcoming FOMC announcements are more informative, it is more likely the insider would acquire private information before FOMC.

5.2 The absence of the pre-FOMC announcement drift in fixed income instruments

In this section, I show that my model can explain the apparent lack of a pre-FOMC announcement drift in fixed income instruments, documented in Lucca and Moench (2015). In the model, there is a positive (negative) pre-announcement drift if the risk exposure β of the asset to the underlying fundamental is positive (negative). In other words, the sign of the pre-FOMC announcement drift in the model depends on whether the asset is risky or a hedge.

Campbell, Sunderam, and Viceira (2017) and Campbell, Pflueger, and Viceira (2020) document that nominal Treasury bonds changed from risky (positively correlated with stocks) in the 1980s and

[19] Starting in January 2019, the chair of the Federal Reserve has held a press conference after each meeting.

1990s to safe (negatively correlated with stocks) in the first decade of the 2000s. The average of the time-varying betas of nominal bonds is close to zero from 1996 to 2019, which results in the absence of a pre-FOMC announcement drift in fixed income instruments. My results agree with Cieslak and Pang (2020), which find that the reduction in the common premium is offset by a decline in the value of the hedging premium, making the overall bond market response economically small and statistically insignificant on FOMC days.

5.3 Risk reduction explanation before FOMC announcements

My model predicts that a more substantial uncertainty reduction is associated with a stronger pre-FOMC announcement drift. In Figure 2, I already showed that the pre-FOMC announcement drift only exists when there is an uncertainty reduction before announcements. To more formally assess the impact of an uncertainty reduction on the excess stock market returns prior to FOMC announcements, I run the following regression:

$$\text{Cum. Return}_t = \alpha + \beta \Delta \text{VIX}_t + \varepsilon_t,$$

where both ΔVIX_t and Cum. Return_t are calculated from 2 p.m on the pre-announcement date to announcement time windows, and t represents each FOMC announcement. As shown in Table 5, when VIX decreases 1% before FOMC news, the cumulative return increases 51.3 basis points on average.[20] In terms of the high-reduction group, since the constant term α is not significantly different from zero, the single variable uncertainty reduction can fully account for the pre-FOMC announcement drift, which is consistent with my model.

The information channel I emphasize is consistent with recent work by Nakamura and Steinsson (2018) and Jarocinski and Karadi (2020). They find that Federal Reserve announcements affect beliefs not only about monetary policy but also about economic fundamentals. Both of the two measures of monetary policy surprises constructed by Nakamura and Steinsson (2018) are indifferent from zero on average. Cieslak and Pang (2020) also show that the average growth news component is close to

[20] This finding is consistent with the simple dummy variable regression model in Table 6, which indicates that the change in VIX before announcements itself can explain a large fraction of the pre-announcement drift.

zero and conclude that FOMC days are not associated with systematically positive or negative news about the economy. Therefore, the pre-FOMC announcement drift cannot be driven by unexpectedly good news.

6 Conclusion

In this paper, I study the private information explanation for the timing and time series pattern of the pre-FOMC announcement drift through informed trading. I extend Kyle's (1985) model such that market makers require compensation for the riskiness of assets' fundamentals prior to FOMC announcements. Informed trading reveals private information and resolves uncertainty gradually, which results in an upward drift in market prices. I demonstrate a strictly positive pre-FOMC announcement drift if and only if market makers are risk compensated. Informed trading is positive on average in order to chase the positive premium, consistent with the data. In addition, the extensions of the benchmark quantitatively account for the timing and time series pattern of the pre-FOMC announcement drift and other important features of the pre-FOMC puzzle.

This paper provides a general framework to account for other pre-event drifts. A large group of papers treat average abnormal positive excess returns before events as evidence of insider trading and tests the market liquidity measure inspired by Kyle (1985), such as Sinha and Gadarowski (2010), Agapova and Madura (2011), and Collin-Dufresne and Fos (2015). In the standard Kyle-type models, however, the expected average excess return before announcements is zero because of risk-neutral market makers. Therefore, this paper provides a general theoretical framework for other pre-event drifts if the risk of news is priced in the pricing kernel. The different equilibrium implications in comparison to the standard Kyle-type models offer new insights into how private news affects asset prices, volatility, volume, and market liquidity. I leave these interesting directions for future work.

Table 1: Summary Statistics on S&P500 Index Excess Returns and Changes in VIX.
Note: This table reports summary statistics for the pre-announcement day 2 p.m (-1) to announcement (ann) and announcement to close. The close time is 3:55 p.m. The samples are: (1) all FOMC announcements, (2 and 3) FOMC announcements sorted on uncertainty, which are the first and third terciles of changes in VIX between open and 2 p.m on pre-announcement dates, and (4 and 5) FOMC announcements with and without an FOMC press conference. The sample period is from 1996:01 to 2019:11, and starting from 2011:04 for the press conference sample. "No. of FOMC" is the number of FOMC announcements in each subset. The t-statistics for the mean are reported in parentheses. ***, **, and * represent 1%, 5%, and 10%.

		(1) All	Sort on Uncertainty		Press Conference	
			(2) High	(3) Low	(4) Yes	(5) No
ΔVIX (%)	2 p.m (-1)-ann	-0.300***	-1.459***	0.748***	-0.734***	-0.072
		(-3.402)	(-14.020)	(5.485)	(-5.213)	(-0.465)
	ann-close	-0.318***	-0.246*	-0.487***	-0.203*	-0.035
		(-4.405)	(-1.819)	(-3.288)	(-1.773)	(-0.181)
	No. of FOMC	187	61	63	34	35
Cum.Return (%)	2 p.m (-1)-ann	0.332***	0.944***	-0.154	0.254***	0.014
		(5.636)	(9.016)	(-1.671)	(3.314)	(0.158)
	ann-close	-0.030	-0.064	0.063	0.099	-0.157
		(-0.446)	(-0.478)	(0.603)	(1.090)	(-1.328)
	No. of FOMC	187	61	63	34	35

Table 2: Comparison of order imbalances conditional on FOMC announcement indicators.
Note: This table compares the level of order imbalances of the E-mini S&P 500 futures in the 24-hour window before FOMC announcements. OIN is the order imbalance defined as $\frac{B-S}{B+S}$, where B (S) is the aggregate buyer-initiated (seller-initiated) trading volume as measured by number of trades. OID is calculated similarly using dollar trading volume. For every FOMC announcement, I calculate the average level of order imbalances in the 24-hour window before FOMC announcements with and without an uncertainty reduction. Column (1) reports the average level of order imbalances on announcements with the pre-FOMC uncertainty reduction ($UR = \pm 1$). Column (2) reports the average level of order imbalances on announcements without the pre-FOMC uncertainty reduction ($UR = 0$). Column (3) reports the difference between columns (1) and (2). The sample period is from 1996:01 to 2019:11. The t-statistics are reported in parentheses. ***, **, and * represent 1%, 5%, and 10%.

	$UR = \pm 1$	$UR = 0$	Difference
	(1)	(2)	(3)
$OIN(\%)$	1.272***	-0.719***	1.991***
	(4.22)	(-2.91)	(5.11)
$OID(\%)$	1.789***	-1.056***	2.845***
	(4.21)	(-3.13)	(5.24)

Table 3: Order imbalances conditional on FOMC announcement indicators.

Note: This table reports ordinary least squares estimates of the relation between event-time order imbalances in the E-mini S&P 500 futures market and announcement day indicators. For each FOMC announcement, the sample includes the announcement day ($ANN = 1$) and non-announcement days in the prior 21 trading days or since the last announcement ($ANN = 0$). OIN is the order imbalance defined as $\frac{B-S}{B+S}$, where B (S) is the aggregate buyer-initiated (seller-initiated) trading volume as measured by number of trades. OID is calculated similarly using dollar trading volume. Both dependent variables are calculated in three event windows: [-24H, 0], [-24H, -12H], and [-12H, 0], where zero is the official release time of the FOMC announcement and the time unit is an hour. The uncertainty-reduced indicator, UR, is equal to one (negative one) for announcements that the pre-FOMC realized return is positive (negative) under an uncertainty reduction before FOMC announcements and zero otherwise. The sample period is from 1996:01 to 2019:11. The t-statistics are reported in parentheses. ***, **, and * represent 1%, 5%, and 10%.

	[-24H, 0]		[-24H, -12H]		[-12H, 0]	
	(1) OIN	(2) OID	(3) OIN	(4) OID	(5) OIN	(6) OID
Constant	-0.047	-0.154**	0.210**	-0.065	-0.104	-0.182**
	(-0.98)	(-2.29)	(2.23)	(-0.49)	(-1.52)	(-1.98)
ANN	-0.222	-0.073	-0.562	-0.319	-0.172	-0.019
	(-0.87)	(-0.20)	(-1.11)	(-0.45)	(-0.47)	(-0.04)
UR	1.851***	2.167***	2.097***	2.213**	1.694***	2.052***
	(5.61)	(4.60)	(3.20)	(2.39)	(3.55)	(3.21)

Table 4: Parameters

The model is calibrated at an annual frequency. I assume that pre-scheduled announcements happen at a monthly frequency.

Parameter	symbol	value
Aggregate output		
long-run output growth rate	$\bar{m}$	1.50%
volatility of aggregate endowment	σ_Y	3.16%
persistence of the AR(1) process	a_m	4.5%
volatility of the AR(1) process	$\sigma_{m,t} \equiv \sigma_{m,0}$	0.4%
Uncertainty and asset value		
transparency of announcements	σ_s	0.45%
exposure of the risky asset	β	3
Preference		
risk aversion	γ	6.6
elasticity of intertemporal substitution	ψ	1
subjective discount factor	ρ	0.005
Other Parameters		
prior of the probability that the insider is informed	π_{nT-1}	0.2
fraction of announcements that the insider is informed	η	0.58
volatility of noise traders	σ_z	17.9%

Table 5: Returns on the S&P 500 Index.

Note: This table shows results for regressing the changes in VIX (ΔVIX) on the cumulative excess returns on the S&P 500 (*Cum.Return*), Cum. $\text{Return}_t = \alpha + \beta \Delta \text{VIX}_t + \varepsilon_t$ where both ΔVIX_t and Cum.Return_t are calculated from 2 p.m on the pre-announcement date to 2 p.m on the announcement date windows, and t represents each FOMC announcement. The samples are: (1) FOMC announcements, (2 and 3) FOMC announcements sorted on uncertainty, which are first and third terciles of changes in VIX (ΔVIX_{t-1}) between open and 2 p.m on pre-announcement dates, and (4 and 5) FOMC announcements with and without an FOMC press conference. The sample period is from 1996:01 to 2019:11, and starting from 2011:04 for the press conference sample. "Obs." and "No. of FOMC" are the number of observations and number of FOMC announcements in each subset, respectively. The t-statistics are reported in parentheses. ***, **, and * represent 1%, 5%, and 10%.

	(1) All	Sort on Uncertianty		Press Conference	
		(2) High	(3) Low	(4) Yes	(5) No
ΔVIX	-0.513***	-0.603***	-0.493***	-0.346***	-0.460***
	(-16.387)	(-5.775)	(-8.405)	(-4.646)	(-7.672)
Constant	0.170***	0.056	0.207***	-0.002	-0.021
	(4.374)	(0.319)	(2.700)	(-0.031)	(-0.390)
Obs.	187	61	63	34	35
No. of FOMC	187	61	63	34	35

Table 6: S&P 500 Index Return Time-Series Regressions.

Note: This table reports results for regressions of the time-series of pre-FOMC announcement returns on various explanatory variables for the sample period 1996:01 to 2019:11. The dependent variable is a time series of cumulative excess returns on the S&P 500 from 2 p.m on the days before announcements to 2 p.m on the days of scheduled FOMC announcements. The first independent variable in columns (1) and (2) is Pre-FOMC dummy (D_{FOMC}), which is equal to one when a scheduled FOMC announcement has been released in the following 24-hour interval and zero otherwise. The second independent variable in column (2) is the interaction of changes in VIX and the pre-FOMC dummy ($\Delta\text{VIX} \times D_{\text{FOMC}}$). "Sharpe ratio" is the annualized Sharpe ratio on FOMC announcement returns. "Obs." and "No. of FOMC" are the number of observations and number of FOMC announcements in each subset, respectively. The t-statistics are reported in parentheses. ***, **, and * represent 1%, 5%, and 10%.

	(1)	(2)
D_{FOMC}	0.314***	0.160**
	(3.737)	(1.857)
$\Delta\text{VIX} \times D_{\text{FOMC}}$		-0.513***
		(-7.507)
Constant	0.010	0.010
	(0.671)	(0.674)
Sharpe Ratio	1.14	1.14
Obs.	5899	5899
No. of FOMC	187	187

APPENDICES

The following appendices provide details of the proof in section 3. Appendix A shows the announcement SDF under recursive utility. Appendix B contains all the proofs for the benchmark economy that the insider is always informed. Other proof is in the online appendix.

A Proof of announcement SDF under recursive utility

A.1 Preferences and the SDF

I assume that the representative agent is endowed with a Kreps-Porteus preference with risk aversion γ and intertemporal elasticity of substitution ψ. In continuous time, the preference is represented by a stochastic differential utility, which can be specified by a pair of aggregators $(f, \mathcal{A})$ such that in the interior of $(nT, (n+1)\,T)$,

$$dV_t = [-f(Y_t, V_t) - \frac{1}{2}\mathcal{A}(V_t)||\sigma_V(t)||^2]dt + \sigma_V(t)dB_t \tag{A1}$$

I adopt the convenient normalization $\mathcal{A}(v) = 0$ and denote $\bar{f}$ the normalized aggregator. Under this normalization, $\bar{f}(C, V)$ is:

$$\bar{f}(C, V) = \frac{\rho}{1 - 1/\psi} \frac{C^{1-1/\psi} - ((1-\gamma)\,V)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)\,V)^{\frac{1-1/\psi}{1-\gamma}-1}}. \tag{A2}$$

The case of $\psi = 1$ is obtained as the limit of (A2) with $\psi \to 1$:

$$\bar{f}(C, V) = \rho V\left[(1-\gamma)\log C - \log\left[(1-\gamma)\,V\right]\right].$$

Because announcements typically result in discrete jumps in the posterior belief about m_t, the value function is typically not continuous at announcements. Given our normalization of the utility function, for $t = nT$, the pre-announcement utility and post-announcement utility are related by:

$$V_t^- = \mathbb{E}_t^-\left[V_t^+\right],$$

where $\mathbb{E}_t^-$ represents expectation with respect to the pre-announcement information at time t.

In the above setup, I can show that the value function of the representative agent takes the form

$$V\left(\hat{m}, t, Y_t\right) = \frac{1}{1-\gamma} H\left(\hat{m}, t\right) Y_t^{1-\gamma},$$

for some twice continuously differentiable function $H\left(\hat{m}, t\right)$. Given the utility of the representative agent, the state price density, denoted $\{\pi_t\}_{t=0}^{\infty}$ can be characterized by the following lemma.

Lemma 4. *For $n = 1, 2, 3\cdots$, in the interior of $((n-1)\,T, nT)$, π_t is a continuous diffusion process with the law of motion*

$$\frac{d\pi_t}{\pi_t} = -r\left(\hat{m}, t\right)dt - \sigma_\pi\left(\hat{m}, t\right)d\tilde{B}_{Y,t},$$

where $r\left(\hat{m}, t\right)$ is the instantaneous risk-free interest rate and $\sigma_\pi\left(\hat{m}, t\right)$ is the market price of risk. At announcements, $t = nT$, π_t is discontinuous, and the announcement stochastic discount factor (A-SDF) is given by

$$\Lambda^*_{t,t+\triangle} = \frac{\left[H\left(\hat{m}_{t+\Delta}, t+\Delta\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}{\left[\mathbb{E}_t\left(H\left(\hat{m}_{t+\Delta}, t+\Delta\right)\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}. \tag{A3}$$

For convenience, I focus on unit IES $\psi = 1$, which results in

$$H(\hat{m}_t, t) = e^{-\frac{\gamma-1}{a_m+\rho}\hat{m}_t + \mathcal{H}(t)} \equiv e^{-\gamma^A \hat{m}_t + \mathcal{H}(t)},$$

where $\gamma^A \equiv \frac{\gamma-1}{a_m+\rho}$.[21] *Moreover, the A-SDF $\Lambda^*_{t,t+\triangle}$ is countercyclical if and only if the agent has early resolution of uncertainty, i.e., $\gamma > \frac{1}{\psi}$, which is equivalent to $\gamma^A > 0$ when $\psi = 1$.*

The proof is in section A of online appendix.

Lemma 5. *Under the assumption that the aggregate endowment does not change in the 24-hour window before announcements, at $t = nT - 1$, the agent has a prior that the expected growth rate upon announcements $\hat{m}_{nT}$ is normally distributed $N(\hat{m}_{nT-1}, \Delta Q)$ where $\Delta Q = q_{nT-1} - q_{nT}$.*

Proof. At announcements $t = nT$, the agent updates her belief using Bayes' rule:

$$\hat{m}_{nT} = q_{nT}\left[\frac{1}{\sigma_s^2}s_n + \frac{1}{q_{nT-1}}\hat{m}_{nT-1}\right], \quad \frac{1}{q_{nT}} = \frac{1}{\sigma_s^2} + \frac{1}{q_{nT-1}}, \tag{A4}$$

which implies

$$\mathbb{E}_{nT-1}[\hat{m}_{nT}] = \hat{m}_{nT-1}, \quad Var_{nT-1}[\hat{m}_{nT}] = \left(\frac{q_{nT}}{\sigma_s^2}\right)^2\left(q_{nT-1} + \sigma_s^2\right) = q_{nT-1} - q_{nT}. \tag{A5}$$

Therefore, at $t = nT - 1$, m_{nT} is normally distributed $N(\hat{m}_{nT-1}, \Delta Q)$ where $\Delta Q = q_{nT-1} - q_{nT}$. ■

B Proof of Theorem 1

The proof is in several steps.

B.1 Step 1: Market Maker's Updating

First, I establish that if market makers conjecture that the insider's trading strategy follows equation (11), then the price dynamics equation (16) satisfies market makers' break-even pricing rule given in equation (7).

Proof of Lemma 1. The conjectured trading strategy (11) implies that

$$\begin{aligned}
\theta_t &= \frac{\log[A(\hat{m}_{nT}, nT)] - \mu_P + \gamma^A\beta\Delta Q}{(nT-t)\lambda} - \frac{Y_t}{nT-t} \\
&= \frac{\frac{\beta-\gamma^A}{\beta}\left\{\log[A(\hat{m}_{nT}, nT)] - \mu_P + \gamma^A\beta\Delta Q\right\}}{(nT-t)\left(\frac{\beta-\gamma^A}{\beta}\lambda\right)} - \frac{Y_t}{nT-t} \\
&= \frac{\left(\log[H(\hat{m}_{nT}, nT)\,A(\hat{m}_{nT}, nT)] - \left(\mu_V - \frac{\gamma^A(\beta-\gamma^A)}{\beta^2}\sigma_v^2\right)\right) / \left(\frac{\beta-\gamma^A}{\beta}\lambda\right) - Y_t}{nT-t},
\end{aligned} \tag{A6}$$

[21] The proof in the appendix provides the formula of SDF for a general IES. All the main results hold under the general IES, which are available upon request.

where the last equality comes from $H(\hat{m}_{nT}, nT) = e^{-\gamma^A \hat{m}_{nT} + \mathcal{H}(nT)}$ and $A(\hat{m}_{nT}, nT) = e^{\beta \hat{m}_{nT} + N(nT)}$. Here $\mu_P = \beta \hat{m}_{nT-1} + N(nT)$ and $\mu_V = \left(\beta - \gamma^A\right) \hat{m}_{nT-1} + \mathcal{H}(nT) + N(nT)$.

Therefore, the aggregate trading volume follows

$$dY_t = \theta_t dt + dZ_t = \frac{\left(\log\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)\right] - \left(\mu_V - \frac{\gamma^A\left(\beta-\gamma^A\right)}{\beta^2}\sigma_v^2\right)\right) / \left(\frac{\beta-\gamma^A}{\beta}\lambda\right) - Y_t}{nT - t} dt + \sigma_z dB_t, \tag{A7}$$

where $dZ_t = \sigma_z dB_t$ and $Y_{nT-1} = 0$.

Now let me define the observation and innovation process. Set $Y^*_{nT-1} = 0$ and

$$\begin{aligned} dY_t^* &= \frac{1}{\sigma_z}\left(dY_t + \frac{\left(\mu_V - \frac{\gamma^A\left(\beta-\gamma^A\right)}{\beta^2}\sigma_v^2\right) / \left(\frac{\beta-\gamma^A}{\beta}\lambda\right) + Y_t}{nT - t} dt\right) \\ &= \frac{\log\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)\right]}{\frac{\beta-\gamma^A}{\beta}\sigma_v (nT - t)} dt + dB_t, \end{aligned}$$

where the last equality comes from $\lambda = \frac{\sigma_v}{\sigma_z}$. Because Y_t are observable to market makers, Y^* is also observable. The corresponding innovation process is given by

$$dB_t^* = \frac{\log\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)\right] - \hat{v}_t}{\frac{\beta-\gamma^A}{\beta}\sigma_v (nT - t)} dt + dB_t$$

where

$$\hat{v}_t = \mathbb{E}\left[\log\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)\right] | \mathcal{F}_t^Y\right]. \tag{A8}$$

The Kalman-filter equation implies

$$d\hat{v}_t = \frac{\sum_{v,t}}{\frac{\beta-\gamma^A}{\beta}\sigma_v (nT - t)} dB_t^*, \tag{A9}$$

where

$$\sum\nolimits_{v,t} = Var\left[\log H(\hat{m}_{nT}, nT) v(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y\right], \tag{A10}$$

is the conditional variance of $\log\left[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)\right]$ given market marker's information (on the filtration $\mathcal{F}_t^Y$). The Kalman-filter equation also implies the dynamics of the posterior variance:

$$\frac{1}{\sum_{v,t}} = \frac{1}{\sum_{v,0}} + \int_{nT-1}^{t} \frac{1}{(nT-s)^2\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2} ds = \frac{1}{\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2} + \frac{t-(nT-1)}{(nT-t)\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2} = \frac{1}{(nT-t)\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2}, \tag{A11}$$

which implies

$$\sum\nolimits_{v,t} = \left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2 (nT - t). \tag{A12}$$

Thus, the filtering equation (A9) is

$$d\hat{v}_t = \frac{\beta - \gamma^A}{\beta}\sigma_v dB_t^* = \frac{\log\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)\right] - \hat{v}_t}{nT - t}dt + \frac{\beta - \gamma^A}{\beta}\sigma_v dB_t. \tag{A13}$$

Define an adjusted order flow $\hat{Y}_t$ as

$$\hat{Y}_t \equiv Y_t - \int_{nT-1}^{t}\left(\frac{\gamma^A \beta \Delta Q}{\lambda}\right)ds = Y_t - \frac{\gamma^A \beta \Delta Q}{\lambda}\left[t - (nT - 1)\right]. \tag{A14}$$

From the aggregate trading volume (A7), the adjusted order flow follows

$$\begin{aligned} d\hat{Y}_t &= dY_t - \frac{\gamma^A \beta \Delta Q}{\lambda}dt \\ &= \frac{\left(\log\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)\right] - \mu_V\right)/\left(\frac{\beta-\gamma^A}{\beta}\lambda\right) - \hat{Y}_t}{nT - t}dt + \sigma_z dB_t. \end{aligned} \tag{A15}$$

This implies

$$\frac{\beta - \gamma^A}{\beta}\lambda d\hat{Y}_t = \frac{\log\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)\right] - \left[\mu_V + \frac{\beta-\gamma^A}{\beta}\lambda\hat{Y}_t\right]}{nT - t}dt + \frac{\beta - \gamma^A}{\beta}\sigma_v dB_t. \tag{A16}$$

Since $\hat{v}_{nT-1} = \mathbb{E}\left[\log H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_{nT-1}^{Y}\right] = \mu_V$ and $\hat{Y}_{nT-1} = 0$, combining (A13) and (A16) gives,

$$d\hat{v}_t = \frac{\beta - \gamma^A}{\beta}\sigma_v dB_t^* = \frac{\beta - \gamma^A}{\beta}\lambda d\hat{Y}_t = \frac{\beta - \gamma^A}{\beta}\lambda\left[dY_t - \frac{\gamma^A \beta \Delta Q}{\lambda}dt\right], \tag{A17}$$

where the last equality holds due to the definition of the adjusted order flow in equation (A14). From the filtering theory, B_t^* is a standard Brownian Motion with respect to market makers' filtration. Therefore, the adjusted order flow $\hat{Y}_t$ is a Brownian Motion with instant variance σ_z^2 under $\mathcal{F}_t^Y$. This also implies

$$\mathbb{E}\left[\theta_t|\mathcal{F}_t^Y\right] = \frac{\gamma^A \beta \Delta Q}{\lambda}, \tag{A18}$$

is market makers' expectation of the insider's order rate, which is strictly positive when market makers are risk compensated.

market makers' prior about $\log\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)\right]$ at time $nT - 1$ is represented by a normal distribution. The Kalman filter implies the posterior distribution of $\log\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)\right]$ under $\mathcal{F}_t^Y$ is also Gaussian, which is summarized by the posterior mean $\hat{v}_t$ and the posterior variance $\sum_{v,t}$. Therefore, $\forall t \in [nT - 1,\, nT]$, market makers' estimation of $H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)$ is

$$\begin{aligned} V_t &= \mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right] \\ &= \mathbb{E}\left[e^{\log H\left(\hat{m}_{nT}, nT\right)A\left(\hat{m}_{nT}, nT\right)}|\mathcal{F}_t^Y\right] \\ &= e^{\hat{v}_t + \frac{1}{2}\sum_{v,t}} = e^{\hat{v}_t + \frac{1}{2}(nT-t)\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2}. \end{aligned} \tag{A19}$$

Applying Ito's Lemma, from equation (A17), I find

$$\begin{aligned}\frac{dV_t}{V_t} &= \frac{1}{V_t}\left[V_t d\hat{v}_t + \frac{1}{2}V_t\left(d\hat{v}_t\right)^2 - \frac{1}{2}\sigma_v^2 V_t dt\right] \\ &= d\hat{v}_t = \frac{\beta-\gamma^A}{\beta}\lambda d\hat{Y}_t.\end{aligned} \tag{A20}$$

Similarly, I define Λ_t^* as the posterior mean of $\log H\left(\hat{m}_{nT}, nT\right)$ under market makers' information:

$$\begin{aligned}\Lambda_t^* &= \mathbb{E}\left[\log H\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right] \\ &= -\gamma^A \mathbb{E}\left[\hat{m}_{nT}|\mathcal{F}_t^Y\right] + \mathcal{H}\left(nT\right) \\ &= \frac{-\gamma^A}{\beta-\gamma^A}\mathbb{E}\left[\left(\beta-\gamma^A\right)\hat{m}_{nT}|\mathcal{F}_t^Y\right] + \mathcal{H}\left(nT\right) \\ &= \frac{-\gamma^A}{\beta-\gamma^A}\mathbb{E}\left[\log H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right] + \frac{\beta\mathcal{H}\left(nT\right)+\gamma^A N\left(nT\right)}{\beta-\gamma^A} \\ &= \frac{-\gamma^A}{\beta-\gamma^A}\hat{v}_t + \frac{\beta\mathcal{H}\left(nT\right)+\gamma^A N\left(nT\right)}{\beta-\gamma^A}\end{aligned} \tag{A21}$$

where the last equality holds due to equation (A8). It implies

$$d\Lambda_t^* = \frac{-\gamma^A}{\beta-\gamma^A}d\hat{v}_t,$$

with $\Lambda_{nT-1}^* = -\gamma^A\hat{m}_{nT-1} + \mathcal{H}\left(nT\right)$. Therefore, the posterior variance of $\log H\left(\hat{m}_{nT}, nT\right)$ under market makers' information is,

$$\Sigma_{\Lambda^*,t} = Var\left[\log\left[H\left(\hat{m}_{nT}, nT\right)\right]|\mathcal{F}_t^Y\right] = \left(\frac{\gamma^A}{\beta-\gamma^A}\right)^2\Sigma_{v,t} = \left(\frac{\gamma^A}{\beta}\right)^2\left(nT-t\right)\sigma_v^2.$$

The Kalman filter implies the posterior distribution of $\log\left[H\left(\hat{m}_{nT}, nT\right)\right]$ under $\mathcal{F}_t^Y$ is also Gaussian, which is summarized by the posterior mean Λ_t^* and the posterior variance $\Sigma_{\Lambda^*,t}$. Therefore, $\forall t \in [nT-1, nT]$, market makers' estimation of $H\left(\hat{m}_{nT}, nT\right)$ is

$$\Lambda_t = \mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right] = \mathbb{E}\left[e^{\log H\left(\hat{m}_{nT}, nT\right)}|\mathcal{F}_t^Y\right] = e^{\Lambda_t^*+\frac{1}{2}\Sigma_{\Lambda^*,t}} = e^{\Lambda_t^*+\frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\left(nT-t\right)\sigma_v^2}.$$

From Ito's Lemma,

$$\begin{aligned}\frac{d\Lambda_t}{\Lambda_t} &= \frac{1}{\Lambda_t}\left[\Lambda_t d\Lambda_t^* + \frac{1}{2}\Lambda_t\left(d\Lambda_t^*\right)^2 - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2\Lambda_t dt\right] \\ &= d\Lambda_t^* = \frac{-\gamma^A}{\beta-\gamma^A}d\hat{v}_t = -\frac{\gamma^A}{\beta}\lambda d\hat{Y}_t.\end{aligned} \tag{A22}$$

Therefore, both V_t and Λ_t are functions of the adjusted order flow $\hat{Y}_t$. From the definition of price dynamics in equation (8),

$$P_t = \frac{\mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right]}{\mathbb{E}\left[H\left(\hat{m}_{nT}, nT\right)|\mathcal{F}_t^Y\right]} = \frac{V\left(t, \hat{Y}_t\right)}{\Lambda\left(t, \hat{Y}_t\right)},$$

the equilibrium pricing rule is also a function of the adjusted order flow, i.e., $P\left(t, \hat{Y}_t\right)$.

I apply Ito's Lemma to V_t,

$$\frac{dV_t}{V_t} = \frac{d\left(P_t \Lambda_t\right)}{P_t \Lambda_t} = \frac{dP_t}{P_t} + \frac{d\Lambda_t}{\Lambda_t} + \frac{dP_t}{P_t}\frac{d\Lambda_t}{\Lambda_t}. \tag{A23}$$

From equations (A20) and (A22), I find

$$\frac{dP\left(t, \hat{Y}_t\right)}{P\left(t, \hat{Y}_t\right)} = \lambda d\hat{Y}_t + \gamma^A \beta \Delta Q dt, \text{ with } P_{nT-1} = e^{\mu_P - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2}. \tag{A24}$$

Furthermore, from equation (A15), the process $\hat{Y}_t$ is a Brownian bridge with instantaneous variance σ_z^2 with respect to the insider's filtration, terminating at $\left(\log\left[H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)\right] - \mu_V\right) / \left(\frac{\beta-\gamma^A}{\beta}\lambda\right)$ (Karatzas and Shreve (1987)). It satisfies $\hat{Y}_t \to \left(\log\left[H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)\right] - \mu_V\right) / \left(\frac{\beta-\gamma^A}{\beta}\lambda\right)$ with probability 1 as $t \to nT$.[22] This implies the equilibrium price in equation (A24) satisfies :

$$\begin{aligned}
\log P_t &= \log P_{nT-1} + \lambda \hat{Y}_t - \left[\frac{1}{2}\sigma_v^2 - \gamma^A \beta \Delta Q\right]\left(t - (nT-1)\right) \qquad \text{(A25)}\\
&= \log P_{nT-1} + \lambda \hat{Y}_t - \frac{1}{2}\frac{\beta - 2\gamma^A}{\beta}\sigma_v^2 \left(t - (nT-1)\right) \\
&\to \beta \hat{m}_{nT-1} - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 + N(nT) + \frac{1}{2}\left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2 + \beta\left(\hat{m}_{nT} - \hat{m}_{nT-1}\right) - \frac{1}{2}\frac{\beta - 2\gamma^A}{\beta}\sigma_v^2 \\
&\to \beta \hat{m}_{nT} + N(nT) = \log A\left(\hat{m}_{nT}, nT\right).
\end{aligned}$$

almost surely as $t \to nT$ from the insider's information. This is equivalent to $P_t \to A\left(\hat{m}_{nT}, nT\right)$ with probability 1 as $t \to nT$ under the insider's filtration.

■

B.2 Step 2: Insider's Optimal Strategy

Second, I capture the insider's optimal trading strategy when the equilibrium pricing rule is a function of the adjusted order flow, i.e., $P\left(t, \hat{Y}_t\right)$.

Proof of Lemma 2. By Theorem 7.6 in Chapter 5 of Karatzas and Shreve (1991) (Feynman-Kac representation), the value function J defined in equation (22), is a unique solution to the Bellman equation (20) with the terminal condition $J\left(nT, y, A\left(\hat{m}_{nT}, nT\right)\right) = j\left(y, A\left(\hat{m}_{nT}, nT\right)\right)$.

Taking the derivative under the expectation operator yields

$$\begin{aligned}
J_y\left(t, y, A\left(\hat{m}_{nT}, nT\right)\right) &= \mathbb{E}\left[j_y\left(y + \omega_{nT} - \omega_t, A\left(\hat{m}_{nT}, nT\right)\right)\right] \\
&= \mathbb{E}\left[g\left(y + \omega_{nT} - \omega_t\right)\right] - A\left(\hat{m}_{nT}, nT\right) \\
&= P(t, y) - A\left(\hat{m}_{nT}, nT\right),
\end{aligned}$$

[22] The distribution of a Brownian bridge is the same as a Brownian motion conditional on the terminal value being known. $\left(\log\left[H\left(\hat{m}_{nT}, nT\right) A\left(\hat{m}_{nT}, nT\right)\right] - \mu_V\right) / \left(\frac{\beta-\gamma^A}{\beta}\lambda\right)$ is the terminal value of $\hat{Y}_t$, which is normally distributed with mean zero and variance σ_z^2 and is independent of Z. Hence, the distribution of $\hat{Y}_t$, unconditional on the terminal value or Z (i.e., from market makers' filtration), are the distribution of a Brownian motion with variance σ_z^2. This is consistent with what I get from the filtering theory in equation (A17).

which shows $J\left(t,y,A\left(\hat{m}_{nT},nT\right)\right)$ also satisfy equation (19) with $P\left(t,y\right)$ as defined by (21).

■

Proof of Lemma 3. For any trading strategy θ_t, apply Ito's Lemma to the value function,

$$\begin{aligned}J\left(nT,\hat{Y}_{nT},A\left(\hat{m}_{nT},nT\right)\right) &= J\left(nT-1,\hat{Y}_{nT-1},A\left(\hat{m}_{nT},nT\right)\right)+\int_{nT-1}^{nT}\left\{J_t dt+J_y d\hat{Y}_t+\frac{1}{2}J_{yy}\left(d\hat{Y}_t\right)^2\right\}\\ &= J\left(nT-1,\hat{Y}_{nT-1},A\left(\hat{m}_{nT},nT\right)\right)+\int_{nT-1}^{nT}\left\{J_t dt+J_y\left(\left[\theta_t-\hat{\theta}_t\right]dt+dZ_t\right)+\frac{1}{2}\sigma_z^2 J_{yy}\right\}\\ &= J\left(nT-1,\hat{Y}_{nT-1},A\left(\hat{m}_{nT},nT\right)\right)-\int_{nT-1}^{nT}\left(A\left(\hat{m}_{nT},nT\right)-P\left(t,\hat{Y}_t\right)\right)\left(\theta_t dt+dZ_t\right),\end{aligned}$$

where I use equations (19) and (20). I can rearrange this as

$$\begin{aligned}\int_{nT-1}^{nT}\left(A\left(\hat{m}_{nT},nT\right)-P\left(t,\hat{Y}_t\right)\right)\theta_t dt &= J\left(nT-1,\hat{Y}_{nT-1},A\left(\hat{m}_{nT},nT\right)\right)-J\left(nT,\hat{Y}_{nT},A\left(\hat{m}_{nT},nT\right)\right)\\ &\quad -\int_{nT-1}^{nT}\left(A\left(\hat{m}_{nT},nT\right)-P\left(t,\hat{Y}_t\right)\right)dZ_t\end{aligned}$$

The left-hand side is the profit of the insider, and the right-hand side is bounded above by

$$J\left(nT-1,\hat{Y}_{nT-1},A\left(\hat{m}_{nT},nT\right)\right)-\int_{nT-1}^{nT}\left(A\left(\hat{m}_{nT},nT\right)-P\left(t,\hat{Y}_t\right)\right)dZ_t \tag{A26}$$

due to the nonnegativity of $J\left(nT,\hat{Y}_{nT},A\left(\hat{m}_{nT},nT\right)\right)$ in equation (22). The no-double-strategies condition

$$E\int_{nT-1}^{nT}P_t^2 dt<\infty$$

implies that the stochastic integral in (A26) has a zero expectation. Therefore,

$$E\int_{nT-1}^{nT}\left\{\left[A\left(\hat{m}_{nT},nT\right)P\left(t,\hat{Y}_t\right)\right]\theta_t dt\right\}t\leq J\left(nT-1,P_{nT-1},A\left(\hat{m}_{nT},nT\right)\right),$$

with equality if and only if $\hat{Y}_{nT}=g^{-1}\left(A\left(\hat{m}_{nT},nT\right)\right)$, which is equivalent to $P\left(nT,\hat{Y}_{nT}\right)=A\left(\hat{m}_{nT},nT\right)$ from equation (21). Thus, $J\left(nT-1,\hat{Y}_{nT-1},A\left(\hat{m}_{nT},nT\right)\right)$ is an upper bound on the insider's expected profit, conditional on the termination value $A\left(\hat{m}_{nT},nT\right)$, and the upper bound is realized - and the corresponding strategy is consequently optimal - if and only if $P\left(nT,\hat{Y}_{nT}\right)=A\left(\hat{m}_{nT},nT\right)$.

■

Having established these results, finally, I show that the conjectured rule by market makers is indeed consistent with the insider's optimal choice, as stated in Theorem 1.

Proof of Theorem 1. Since $\hat{Y}_{nT}=g^{-1}\left(A\left(\hat{m}_{nT},nT\right)\right)$ a.s., for any scalar a, the probability, given market makers' information at time $nT-1$, that $\hat{Y}_{nT}\leq a$ is $F\left(g\left(A\left(\hat{m}_{nT},nT\right)\right)\right)$ where F is the distribution function of $A\left(\hat{m}_{nT},nT\right)$. According to Lemma , the distribution function of $\hat{Y}_{nT}$, given market makers' information at time 0, is normal distribution with mean zero and variance σ_z^2 and I denote it as N. Therefore, $N=F\circ g$, implying $g=F^{-1}\circ N$. When $\log A\left(\hat{m}_{nT},nT\right)$ is normally distributed with mean $\beta\hat{m}_{nT-1}+N\left(nT\right)$ and variance $\beta^2\left[q_{nT-1}-q_{nT}\right]=\sigma_v^2$. Set $g\left(y\right)=F^{-1}\left(N\left(y\right)\right)$:

$$F\left(g\left(y\right)\right)=N^*\left(\frac{\log g\left(y\right)-\left[\beta\hat{m}_{nT-1}+N\left(nT\right)\right]}{\sigma_v}\right)=N^*\left(\frac{\hat{Y}_t}{\sigma_z}\right),$$

so

$$g(y) = \exp\left(\beta\hat{m}_{nT-1} + N(nT) + \lambda y\right), \tag{A27}$$

where $\lambda = \frac{\sigma_v}{\sigma_z}$ and $g(y)$ is a increasing function in y since $\lambda > 0$. From the conjectured trading strategy in equation (11), $\hat{\theta}_t \equiv \mathbb{E}\left[\theta_t | \mathcal{F}_t^Y\right] = \frac{\gamma^A \beta \Delta Q}{\lambda}$. It implies

$$\begin{aligned}
P\left(t, \hat{Y}_t\right) &= \mathbb{E}\left[g\left(\hat{Y}_t + \omega_{nT} - \omega_t\right)\right] \\
&= \mathbb{E}\left[\exp\left(\beta\hat{m}_{nT-1} + N(nT) + \lambda\left(\hat{Y}_t + Z_{nT} - Z_t - \frac{\gamma^A\beta\Delta Q}{\lambda}(nT - t)\right)\right)\right] \\
&= \exp\left(\beta\hat{m}_{nT-1} + N(nT) + \lambda\hat{Y}_t + \frac{1}{2}\sigma_v^2(nT - t) - \gamma^A\beta\Delta Q(nT - t)\right) \\
&= \exp\left(\log P_{nT-1} + \lambda\hat{Y}_t - \left[\frac{1}{2}\sigma_v^2 - \gamma^A\beta\Delta Q\right](t - (nT - 1))\right)
\end{aligned} \tag{A28}$$

where $P_{nT-1} = e^{\beta\hat{m}_{nT-1} - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2 + N(nT) + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2}$, which is exactly equation (A25). Therefore, the pricing function in equation (A27) implies the price dynamics follow equation (A24).

Equation (21) implies that $P(t, \omega_t)$ is a martingale under the filtration generated by ω. This implies the price dynamics and the expected trading volume $\hat{\theta}(t)$ with respect to $\mathcal{F}_t$ must satisfy

$$P_t - \hat{\theta}(t) P_y + \frac{1}{2}\sigma_z^2 P_{yy} = 0. \tag{A29}$$

It's very straightforward to show this pricing rule in equation (A28) satisfies the above property.

Besides, In the proof of Lemma 1, I have already shown that the trading strategy in (24) implies $P\left(t, \hat{Y}_t\right) \to A\left(\hat{m}_{nT}, nT\right)$ with probability 1 as $t \to nT$. It follows that the strategy (24) is optimal. Therefore, $\{P_t, \theta_t\}$ in equations (23) and (24) is an equilibrium.

Combining equations (22) and (A27), the maximized expected profit of the insider is

$$\begin{aligned}
J\left(t, P\left(t, \hat{Y}_t\right), A\left(\hat{m}_{nT}, nT\right)\right) &= \frac{1}{2}\sigma_v\sigma_z(nT - t) A\left(\hat{m}_{nT}, nT\right) \\
+ \frac{P\left(t, \hat{Y}_t\right) - A\left(\hat{m}_{nT}, nT\right) + A\left(\hat{m}_{nT}, nT\right)\left[\log A\left(\hat{m}_{nT}, nT\right) - \log P\left(t, \hat{Y}_t\right)\right]}{\lambda}&.
\end{aligned} \tag{A30}$$

As in Back (1992), I explicitly indicate the conditional expectation at time t given market makers' information by $E^M[\cdot]$ and the conditional expectation given the insider's information by $E^I[\cdot]$. Given equation (A27), the pricing rule in equation (21) yields

$$\begin{aligned}
P(t, Z_t) &= E^I\left[g\left(Z_t + \omega_{nT} - \omega_t\right) | Z_t\right] \\
&\overset{(A27)}{=} E^I\left[\exp\left(\beta\hat{m}_{nT-1} + N(nT) + \lambda\left(Z_{nT} - \frac{\gamma^A\beta\Delta Q}{\lambda}(nT - t)\right)\right) | Z_t\right] \\
&= E^I\left[P(nT, Z_{nT}) | Z_t\right]\exp\left(-\gamma^A\beta\Delta Q(nT - t)\right),
\end{aligned} \tag{A31}$$

where the last equality comes from equation (A28) when $t = nT$:

$$\begin{aligned}
P(nT, Z_{nT}) &= \exp\left(\log P_{nT-1} + \lambda Z_{nT} - \left[\frac{1}{2}\sigma_v^2 - \gamma^A\beta\Delta Q\right]\right) \\
&= \exp\left(\beta\hat{m}_{nT-1} + N(nT) + \lambda Z_{nT}\right).
\end{aligned}$$

Rearrange equation (A31), I find

$$P\left(t,Z_t\right)\exp\left(-\gamma^A\beta\Delta Q\left(t-\left(nT-1\right)\right)\right)=E^I\left[P\left(nT,Z_{nT}\right)|Z_t\right]\exp\left(-\gamma^A\beta\Delta Q\right),$$

which implies $P\left(t,Z_t\right)\exp\left(-\gamma^A\beta\Delta Q\left(t-\left(nT-1\right)\right)\right)$ is a martingale under the insider's information set. Since the distribution of Z_t with respect to the insider's information is the same as the distribution of $\hat{Y}_t$ with respect to the market makers' information,

$$\begin{aligned}P\left(t,\hat{Y}_t\right)\exp\left(-\gamma^A\beta\Delta Q\left(t-\left(nT-1\right)\right)\right)&=E^M\left[P\left(nT,\hat{Y}_{nT}\right)|\hat{Y}_t\right]\exp\left(-\gamma^A\beta\Delta Q\right)\\&=E^M\left[P\left(nT,\hat{Y}_{nT}\right)|\left(\hat{Y}_s\right)_{s\leq t}\right]\exp\left(-\gamma^A\beta\Delta Q\right)\end{aligned}$$

where the last equality using the Markov property of a Brownian motion. This implies

$$P\left(t,\hat{Y}_t\right)\exp\left(-\gamma^A\beta\Delta Q\left(t-\left(nT-1\right)\right)\right)$$

is a martingale under the insider's information set. This is equivalent to say $P\left(t,\hat{Y}_t\right)$ is a submartingale with a deterministic growth rate $\gamma^A\beta\Delta Q$ per unit of time since both γ^A and β are strictly positive. Similar argument applies to the price-response function $P_{\hat{Y}}\left(t,\hat{Y}\right)$, which is also a submartingale with a deterministic growth rate $\gamma^A\beta\Delta Q$ per unit of time.

Therefore, the unconditional expected return for any $t\in[nT-1,\,nT]$ is

$$\log\mathbb{E}\left[\frac{P_t}{P_{nT-1}}|\mathcal{F}^Y_{nT-1}\right]=\gamma^A\beta\Delta Q\left(t-\left(nT-1\right)\right)=\gamma^A\beta\Delta Q\left(t-\left(nT-1\right)\right),\tag{A32}$$

which implies the expected pre-FOMC announcement drift grows at a constant rate $\gamma^A\beta\Delta Q$. ■

Next, I prove the properties of the equilibrium in Theorem 1.

Proof of Proposition 1. In the meantime, the posterior variance of $\log P_{nT}$ at time $t\in[nT-1,\,nT]$ is

$$\begin{aligned}Var\left[\log P_{nT}|\mathcal{F}^Y_t\right]&=Var\left[\log A\left(\hat{m}_{nT},nT\right)|\mathcal{F}^Y_t\right]\\&=Var\left\{\log\left[H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)\right]-\log\left[H\left(\hat{m}_{nT},nT\right)\right]|\mathcal{F}^Y_t\right\}\\&=Var\left\{\log\left[H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)\right]|\mathcal{F}^Y_t\right\}+Var\left\{\log\left[H\left(\hat{m}_{nT},nT\right)\right]|\mathcal{F}^Y_t\right\}\\&\quad-2Cov\left(\log\left[H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)\right],\log\left[H\left(\hat{m}_{nT},nT\right)\right]|\mathcal{F}^Y_t\right)\\&=\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2\left(nT-t\right)-2\frac{-\gamma^A\left(\beta-\gamma^A\right)}{\beta^2}\sigma_v^2\left(nT-t\right)+\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2\left(nT-t\right)\\&=\sigma_v^2\left(nT-t\right)=\beta^2\Delta Q\left(nT-t\right).\end{aligned}$$

Therefore, the reduction in uncertainty at time t comparing to $nT-1$ is

$$Var\left[\log P_{nT}|\mathcal{F}^Y_t\right]-Var\left[\log P_{nT}|\mathcal{F}^Y_{nT-1}\right]=\beta^2\Delta Q\left[\left(nT-1\right)-t\right],$$

which implies uncertainty reduces at a constant rate $\beta^2\Delta Q$ per unit of time. ■

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The Pre-FOMC Announcement Drift and Private Information: Kyle Meets Macro-Finance

Internet Appendix

This Online Appendix contains additional analysis to accompany the manuscript. Section A proves the proof of Lemma 4. Section B proves the equilibrium when the insider can optimally choose when to start to trade. Section C provides the details for the economy that market makers are uncertain about whether the insider is informed or not.

A Proof of Lemma 4

Proof. The HJB equation for recursive utility satisfies

$$\bar{f}(Y_t, V(\hat{m}_t, t, Y_t)) + \mathcal{L}[(\hat{m}_t, t, Y_t)] = 0.$$

Due to homogeneity, consider the value function of the form

$$V(\hat{m}_t, t, Y_t) = \frac{1}{1-\gamma} H(\hat{m}_t, t) Y_t^{1-\gamma},$$

where $H(\hat{m}_t, t)$ satisfies the following HJB equation:

$$\begin{aligned} 0 \quad = \quad & \frac{\rho}{1-\frac{1}{\psi}}\left(H(\hat{m}_t, t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} - 1\right) + \left(\hat{m}_t - \frac{1}{2}\gamma\sigma_C^2\right) + \frac{1}{1-\gamma}\frac{H_t(\hat{m}_t, t)}{H(\hat{m}_t, t)} \\ & + \left[\frac{1}{1-\gamma}a_m(\bar{m} - \hat{m}_t) + q_t\right]\frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)} + \frac{1}{2}\frac{1}{1-\gamma}\frac{H_{mm}(\hat{m}_t, t)}{H(\hat{m}_t, t)}\left(\frac{q_t}{\sigma_C}\right)^2. \end{aligned} \tag{A.1}$$

with the boundary condition that

$$H\left(\hat{m}_t^-, t^-\right) = \mathbb{E}\left[H\left(\hat{m}_t^+, t^+\right) | \hat{m}_t^-, q_t^-\right]. \tag{A.2}$$

The state price process of recursive utility satisfies

$$\frac{d\pi_t}{\pi_t} = \frac{d\bar{f}_C(Y_t, V_t)}{\bar{f}_C(Y_t, V_t)} + \bar{f}_V(Y_t, V_t)\,dt. \tag{A.3}$$

Therefore, for $n = 1, 2, \cdots$, in the interior of $(nT, (n+1)T)$, the law of motion of the state price density, π_t satisfies the stochastic differential equation of the form:

$$d\pi_t = \pi_t\left[-r(\hat{m}, t)\,dt - \sigma_\pi(\hat{m}, t)\,d\tilde{B}_{Y,t}\right],$$

where

$$r(\hat{m}, t) = \rho + \frac{1}{\psi}\hat{m} - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_C^2 - \frac{\gamma - \frac{1}{\psi}}{1-\gamma}\frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)}q(t) + \frac{1}{2}\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\frac{1-\frac{1}{\psi}}{1-\gamma}\left(\frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)}\right)^2\left(\frac{q(t)}{\sigma_C}\right)^2,$$

is the risk-free interest rate, and

$$\sigma_\pi(\hat{m},t) = \gamma\sigma_C - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_m(\hat{m}_t,t)}{H(\hat{m}_t,t)}\frac{q(t)}{\sigma_C},$$

is the market price of the Brownian motion risk.

Upon announcements, the stochastic discount factor for a small interval $\triangle$ is

$$\text{SDF}_{t,t+\triangle} = e^{-\rho\triangle}\left(\frac{C_{t+\triangle}}{Y_t}\right)^{-\frac{1}{\psi}}\left(\frac{W_{t+\triangle}}{E_t\left[W_{t+\triangle}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}, \tag{A.4}$$

where

$$W_t = \left[(1-\gamma)V(\hat{m},t,C)\right]^{\frac{1}{1-\gamma}}, \tag{A.5}$$

which implies

$$\text{SDF}_{t,t+\triangle} = e^{-\rho\triangle}\left(\frac{C_{t+\triangle}}{Y_t}\right)^{-\frac{1}{\psi}}\left(\frac{\left(H(\hat{m}_{t+\Delta},t+\triangle)C_{t+\Delta}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}{\mathbb{E}_t\left[H(\hat{m}_{t+\Delta},t+\triangle)C_{t+\Delta}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

Therefore, when upon announcements ($t = nT$) and $\triangle \to 0$,

$$\Lambda^*_{t,t+\triangle} = \frac{\left[H(\hat{m}_{t+\Delta},t+\Delta)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}{\left[\mathbb{E}_t\left(H(\hat{m}_{t+\Delta},t+\Delta)\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}. \tag{A.6}$$

The term $\beta H(\hat{m}_t,t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}$ is the endowment-wealth ratio. Consider the following log-linear expansion: $e^{\ln x} \approx e^{\ln \bar{x}} + e^{\ln \bar{x}}(\ln x - \ln \bar{x})$,

$$\beta H(\hat{m}_t,t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} \approx \kappa + \kappa\left[\ln\rho - \frac{1-\frac{1}{\psi}}{1-\gamma}\ln H(\hat{m}_t,t) - \ln\kappa\right],$$

where $\kappa = \beta H(\bar{m},t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}$ is the endowment-wealth ratio when $\hat{m}_t$ is equal to its unconditional mean $\bar{m}$.

Therefore, I can approximate $\beta H\left(\hat{m}_t,t\right)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}$ as

$$\begin{aligned}\frac{\rho}{1-\frac{1}{\psi}}\left[H\left(\hat{m}_t,t\right)^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}}-1\right] &\approx \frac{1}{1-\frac{1}{\psi}}\left[\kappa+\kappa\left[\ln\rho-\frac{1-\frac{1}{\psi}}{1-\gamma}\ln H\left(\hat{m}_t,t\right)-\ln\kappa\right]-\rho\right] \\ &= -\frac{\kappa}{1-\gamma}\ln H\left(\hat{m}_t,t\right)+\xi_0,\end{aligned}$$

where I denote $\xi_0 \overset{\triangle}{=} \frac{1}{1-\frac{1}{\psi}}\left[\kappa-\rho-\kappa\left(\ln\kappa-\ln\rho\right)\right]$.

The HJB equation (A.1) is written as

$$\begin{aligned}&\xi_0-\frac{\kappa}{1-\gamma}\ln H\left(\hat{m}_t,t\right)+\left(\hat{m}_t-\frac{1}{2}\gamma\sigma_C^2\right)+\frac{1}{1-\gamma}\frac{H_t\left(\hat{m}_t,t\right)}{H\left(\hat{m}_t,t\right)} \\ &+\left[\frac{1}{1-\gamma}a_m\left(\bar{m}-\hat{m}_t\right)+q_t\right]\frac{H_m\left(\hat{m}_t,t\right)}{H\left(\hat{m}_t,t\right)}+\frac{1}{2}\frac{1}{1-\gamma}\frac{H_{mm}\left(\hat{m}_t,t\right)}{H\left(\hat{m}_t,t\right)}\left(\frac{q_t}{\sigma_C}\right)^2=0.\end{aligned} \tag{A.7}$$

I guess $H\left(\hat{m}_t,t\right)$ is of the form

$$H\left(\hat{m}_t,t\right)=e^{-\gamma^A\hat{m}_t+\mathcal{H}(t)}. \tag{A.8}$$

Using the method of undetermined coefficients, I have that

$$\gamma^A = \frac{\gamma-1}{a_m+\kappa}, \tag{A.9}$$

$$\mathcal{H}'\left(t\right) = \kappa\mathcal{H}\left(t\right)-f\left(t\right). \tag{A.10}$$

where $f\left(t\right)$ is defined as:

$$f\left(t\right)=\frac{\left(1-\gamma\right)^2}{a_m+\kappa}q\left(t\right)+\frac{1}{2}\frac{\left(1-\gamma\right)^2}{\left(a_m+\kappa\right)^2}\frac{1}{\sigma_C^2}q^2\left(t\right)-\frac{1}{2}\gamma\left(1-\gamma\right)\sigma_C^2+a_m\bar{m}\frac{1-\gamma}{a_m+\kappa}+\xi_0.$$

$\mathcal{H}\left(t\right)$ can be solved in closed form from equations (A.10) and (A.2). In order to solve for asset prices, I do not need the functional form $\mathcal{H}\left(t\right)$.

Note that this above approximation is *exact* if $\psi=1$, in which case

$$\gamma^A=\frac{\gamma-1}{a_m+\rho}. \tag{A.11}$$

Besides, from equations (A.6) and (A.9), it is straightforward to show the A-SDF is countercyclical if and only if the agent has early resolution of uncertainty, i.e., $\gamma>\frac{1}{\psi}$, which is equivalent to $\gamma^A>0$ when $\psi=1$. ■

B Proof of Theorem 2

The proof of equilibrium price and optimal strategy is similar to that in Theorem 1. I focus on the expected profit when the insider can choose when to start to trade.

$\forall t \in [s, nT)$, the expected profit under a final asset value $A(\hat{m}_{nT}, nT)$ is

$$\begin{aligned}
J\left(t, P\left(t, \hat{Y}_t\right), A\left(\hat{m}_{nT}, nT\right)\right) &= E\left[j\left(\omega_{nT}, A\left(\hat{m}_{nT}, nT\right)\right)|\omega_t = \hat{Y}_t\right] \\
&= E\left[\int_{\hat{Y}_t+\omega_{nT}-\omega_t}^{\frac{\log A(\hat{m}_{nT}, nT)-\mu_P}{\lambda_s}} \left(A\left(\hat{m}_{nT}, nT\right) - e^{\mu_P+\lambda_s x}\right) dx\right] \\
&= A\left(\hat{m}_{nT}, nT\right) \frac{\log A\left(\hat{m}_{nT}, nT\right) - \mu_P}{\lambda_s} - \frac{A\left(\hat{m}_{nT}, nT\right)}{\lambda_s} - A\left(\hat{m}_{nT}, nT\right)\hat{Y}_t \\
&\quad + A\left(\hat{m}_{nT}, nT\right) \frac{\gamma^A \beta \triangle Q_s}{(nT-s)\lambda_s}(nT-t) + \frac{1}{\lambda_s} e^{\mu_P+\lambda_s \hat{Y}_t + \frac{1}{2}\sigma_v^2(nT-t) - \frac{\gamma^A\beta\triangle Q_s}{(nT-s)}(nT-t)} \\
&= \frac{A\left(\hat{m}_{nT}, nT\right)\left[\log A\left(\hat{m}_{nT}, nT\right) - \log P_t\right] - \left[A\left(\hat{m}_{nT}, nT\right) - P_t\right]}{\lambda_s} + \frac{1}{2} A\left(\hat{m}_{nT}, nT\right)\sigma_v\sigma_z\left(nT-t\right).
\end{aligned}$$

Therefore, the unconditional expected profits at $s \in [(n-1)T, nT)$ is

$$\begin{aligned}
&E\left[J\left(s, P\left(s, \hat{Y}_s\right), A\left(\hat{m}_{nT}, nT\right)\right)\right] \\
=&E\left[\frac{A\left(\hat{m}_{nT}, nT\right)\left[\log A\left(\hat{m}_{nT}, nT\right) - \log P\left(s, \hat{Y}_s\right)\right] - \left[A\left(\hat{m}_{nT}, nT\right) - P\left(s, \hat{Y}_s\right)\right]}{\lambda_s} + \frac{1}{2} A\left(\hat{m}_{nT}, nT\right)\sigma_v\sigma_z\left(nT-s\right)\right] \\
=&\frac{e^{\mu_{P,s}+\frac{1}{2}\Sigma_s}}{\lambda_s}\left(\frac{\beta+\gamma^A}{\beta}\Sigma_s - 1 + e^{-\frac{\gamma^A}{\beta}\Sigma_s}\right).
\end{aligned}$$

When $s = (n-1)T$, $\Sigma_{(n-1)T} = \beta^2[q_{(n-1)T} - q_{nT}] = 0$ so that the unconditional expected profit is zero. When $s \to nT^-$, $\Sigma_{nT^-} = \beta^2[q_{nT^-} - q_{nT}]$ is finite and the price impact $\lambda_s = \frac{\sigma_{v,s}}{\sigma_z} = \frac{\sqrt{\frac{\Sigma_s}{nT-s}}}{\sigma_z}$ goes to infinite, which implies the unconditional expected profit converges to zero.

C Proof of Theorem 3

The proof is in several steps. At the beginning, I provide the essential tools to construct the equilibrium of the model.[1]

[1]The method of proof is based on Li (2013) that solves the economy with risk-neutral market makers. He applies the "sequential detection" in the filtering literature.

C.1 Step 0: tools for market makers' updating

Lemma 6. *Let $\mu(t, V)$ be the estimate of the unnormalized density function of the random variable $V = H(\hat{m}_{nT}, nT)\, A(\hat{m}_{nT}, nT)$ given the stochastic differential equation (35) when the insider is informed. Then $\mu(t, V)$ must satisfy the following stochastic differential equation (Zakai equation):*

$$d\mu(t, V) = \frac{\theta\left(t, \tilde{V}\right)}{\sigma_z^2} \mu(t, V)\, dY_t, \qquad \mu(0, V) = f(V),$$

which has a unique solution

$$\mu(t, V) = f(V) \exp\left[\frac{1}{\sigma_z^2}\left(\int_0^t \theta(s, V)\, dY(s) - \frac{1}{2}\int_0^t \theta^2(s, V)\, ds\right)\right].$$

Hence, the value estimate $V(t)$ of $H(\hat{m}_{nT}, nT)\, A(\hat{m}_{nT}, nT)$ is given by

$$V(t) \equiv \mathbb{E}\left[H(\hat{m}_{nT}, nT)\, A(\hat{m}_{nT}, nT) \,|\mathcal{F}_{1,t}\right] = \frac{\int_{\mathrm{V}} V \mu(V, t)\, dV}{\int_{\mathrm{V}} \mu(V, t)\, dV} \tag{C.12}$$

where $f(v) = \frac{dF(v)}{dv}$ is the prior probability density function at time 0.

Proof. See Zakai (1969) or Baras (1991). ■

Lemma 7. *The value estimate given by (C.12) satisfies the stochastic differential equation*

$$dV(t) = \lambda(t)\left(dY_t - \hat{\theta}(t)\, dt\right), \tag{C.13}$$

where

$$\hat{\theta}(t) = \mathbb{E}\left[\theta\left(t, \tilde{V}\right)|\mathcal{F}_{1,t}\right] = \frac{\int_{\mathrm{V}} \theta\left(t, \tilde{V}\right) \mu(V, t)\, dv}{\int_{\mathrm{V}} \mu(V, t)\, dv} \tag{C.14}$$

and

$$\lambda(t) \equiv \frac{\mathbb{E}\left[\theta\left(t, \tilde{V}\right)|\mathcal{F}_{1,t}\right] - V(t)\, \hat{\theta}(t)}{\sigma_z^2}. \tag{C.15}$$

In addition,

$$\hat{Y}_{1,t} \equiv Y_t - \int_0^t \hat{\theta}(s)\, ds \tag{C.16}$$

is a Brownian Motion with instant variance σ_z^2 under $\mathcal{F}_{1,t}$.

Proof. Applying Ito's Lemma to equation (C.12) leads to the above standard filtering results. ■

Through observing the aggregate trading volume Y_t, market makers estimate the probability that Y_t is generated by the the insider has private information or not. This updating problem can be solved as to calculate the likelihood ratio between the two hypotheses, $\delta = 1$ versus $\delta = 0$. Following Li (2013), the logarithm of the likelihood ratio between hypotheses (35) and (36) is given by

$$\phi(t) \equiv \frac{1}{\sigma_z^2}\left(\int_0^t \left[\hat{\theta}(s) - \theta(s, \bar{V})\right] dY(s) - \frac{1}{2}\int_0^t \left[\hat{\theta}^2(s) - \theta^2(s, \bar{V})\right] ds\right)$$

where $\hat{\theta}$ is as defined by (C.14).

Lemma 8. *market makers' estimate of the probability that the strategic trader has private information*

$$\pi(t) = \mathbb{E}\left[\delta | \mathcal{F}_t^Y\right] = \frac{\pi_{nT-1}\exp[\phi(t)]}{1 - \pi_{nT-1} + \pi_{nT-1}\exp[\phi(t)]} \tag{C.17}$$

satisfies the following stochastic differential equation:

$$d\pi(t) = \frac{\pi(t)[1-\pi(t)]}{\sigma_z^2}\left(\hat{\theta}(t) - \theta(t, \bar{V})\right) d\hat{Y}(t), \qquad \pi(0) = \pi_{nT-1} \tag{C.18}$$

where

$$\hat{Y}(t) = Y_t - \int_0^t \left(\pi(s)\hat{\theta}(s) + [1-\pi(s)]\theta(s, \bar{V})\right) ds \tag{C.19}$$

is the information process, which is a Brownian motion with instantaneous variance σ_z^2 under the filtration $\mathcal{F}_t^Y$.

Proof. The definition of $\pi(t)$ in equation (C.17) is obtained by the Bayes' rule. By Ito's Lemma,

$$\begin{aligned} d\pi(t) &= \pi_\phi d\phi + \frac{1}{2}\pi_{\phi\phi}(d\phi)^2 \\ &= \pi(1-\pi)\, d\phi + \frac{1}{2}\pi(1-\pi)(1-2\pi)(d\phi)^2 \\ &= \frac{\pi(t)[1-\pi(t)]}{\sigma_z^2}\left(\hat{\theta}(t) - \theta(t, \bar{V})\right) d\hat{Y}(t) \end{aligned}$$

where

$$\hat{Y}(t) = Y_t - \int_0^t \left(\pi(s)\hat{\theta}(s) + [1-\pi(s)]\theta(s, \bar{V})\right) ds.$$

■

Lemma 8 shows that market makers' probability estimate is governed by a nonlinear stochastic differential equation. Note that when the prior $\pi_{nT-1} = 0$ or $\pi_{nT-1} = 1$, the solution to the belief dynamics (C.18) is $\pi(t) \equiv 0$ or $\pi(t) \equiv 1$, respectively.

C.2 Step 1: market makers' updating

First, I show that given the insider's trading strategies when she is informed and not informed, how market makers estimate the probability that the insider has private information and the price dynamics through nonlinear filtering.

Let $\Pi\left(t,y\right)$ be an arbitrary function in $C^{1,2}$ on $[0,1]\times R$ with a close range $[0,1]$. At time $nT-1$, $\log\left[H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right.\right.$ is normally distributed with mean μ_v and variance $\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2$. I define $h\left(y\right)=\exp\left(\mu_v+\frac{\beta-\gamma^A}{\beta}\lambda y\right)$ and $\bar{V}=e^{\mu_v+\frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2}$. This implies $h^{-1}\left(\bar{V}\right)=\frac{\frac{\beta-\gamma^A}{\beta}\sigma_v^2}{2\lambda}$.

I guess the insider's trading strategy follows

$$\theta\left(t,y,V\right)=\frac{h^{-1}\left(V\right)-h^{-1}\left(\bar{V}\right)-\Pi\left(t,y\right)\left[y-h^{-1}\left(\bar{V}\right)\right]}{nT-t}+\bar{\theta}\left(t,y\right), \tag{C.20}$$

and

$$\Theta\left(t,y\right)=\frac{\left[1-\Pi\left(t,y\right)\right]\left[y-h^{-1}\left(\bar{V}\right)\right]}{nT-t}+\bar{\theta}\left(t,y\right). \tag{C.21}$$

The expected trading rate of the insider under market makers' perspective $\mathcal{F}_t^Y$ is

$$\bar{\theta}\left(t,y\right)=\frac{\left(\frac{\gamma^A}{\beta}\lambda\sigma_z^2\Pi\left(t,y\right)-\Pi_y\left(t,y\right)\sigma_z^2\right)\cdot E\left(t,y\right)+\Pi_y\left(t,y\right)\sigma_z^2}{\Pi\cdot E\left(t,y\right)+1-\Pi}, \tag{C.22}$$

where I let $E\left(t,y\right)$ be defined as[2]

$$E\left(t,y\right)=e^{-\frac{\gamma^A}{\beta}\lambda y-\frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2\left(t-\left(nT-1\right)\right)}. \tag{C.23}$$

The following Lemma states market makers' expectation of the insider's order rate and their value estimate of $H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)$, given the insider's order rate $\theta\left(t,y,V\right)$ defined in equation (C.20).

Lemma 9. *Let $\hat{Y}_{1,t}$ be a Brownian bridge that satisfies*

$$d\hat{Y}_{1,t}=\left[\theta\left(t,\hat{Y}_{1,t},V\right)-\Theta\left(t,\hat{Y}_{1,t}\right)\right]dt+dZ_t \tag{C.24}$$

$$=\frac{h^{-1}\left(V\right)-\hat{Y}_{1,t}}{nT-t}dt+dZ_t \tag{C.25}$$

with $\hat{Y}_{1,nT-1}=0$. If the insider's order rate is $\theta\left(t,\hat{Y}_{1,t},V\right)$, as defined by (C.20), $\Theta\left(t,\hat{Y}_{1,t}\right)$ as defined by (C.21), is then

[2] As shown later, Li (2013) is a special case of this economy where $\bar{\theta}\left(t,y\right)=0$ when the market makers are risk-neutral.

market makers' expected order rate from the insider, conditional on the insider having private information. That is,

$$\hat{\theta}(t) = \mathbb{E}\left[\theta\left(t, \hat{Y}_{1,t}, V\right)|\mathcal{F}_{1,t}\right] = \Theta\left(t, \hat{Y}_{1,t}\right).$$

Furthermore, the expected value of $H(\hat{m}_{nT}, nT)\, A(\hat{m}_{nT}, nT)$ under $\mathcal{F}_{1,t}$ is

$$V(t) = \mathbb{H}\left(t, \hat{Y}_{1,t}\right),$$

where

$$\mathbb{H}(t, y) = \mathbb{E}\left[h\left(y + Z_{nT} - Z_t\right)\right],$$

where the expectation is taken over the Brownian motion Z.

Proof. See Lemma 6 in Li (2013). ■

From equation (C.18), market makers' estimate of the probability that the insider has private information satisfies

$$d\Pi\left(t, \hat{Y}_{1,t}\right) = \frac{\Pi\left(t, \hat{Y}_{1,t}\right)\left[1 - \Pi\left(t, \hat{Y}_{1,t}\right)\right]}{\sigma_z^2} \frac{\hat{Y}_{1,t} - h^{-1}(\bar{V})}{nT - t} \times \left(d\hat{Y}_{1,t} + \frac{\left[1 - \Pi\left(t, \hat{Y}_{1,t}\right)\right]\left[\hat{Y}_{1,t} - h^{-1}(\bar{V})\right]}{nT - t} dt\right) \tag{C.26}$$

with $\Pi\left(nT-1, \hat{Y}_{1,nT-1}\right) = \pi_{nT-1}$. This equation holds because of Lemma 8 and the following equation:

$$\begin{aligned} dY_t &\overset{(C.19),(C.30)}{=} d\hat{Y}(t) + \bar{\theta}(t, y) \\ &\overset{(C.16)}{=} d\hat{Y}_{1,t} + \Theta(t, y) \\ &\overset{(C.21)}{=} d\hat{Y}_{1,t} + \frac{\left[1 - \Pi(t, y)\right]\left[y - h^{-1}(\bar{V})\right]}{nT - t} + \bar{\theta}(t, y). \end{aligned} \tag{C.27}$$

As shown in Li (2013), $\forall t \in [nT-1,\, nT]$, the solution to the stochastic differential equation (C.26) is:

$$\Pi\left(t, \hat{Y}_{1,t}\right) = \frac{\pi_{nT-1} \exp\left(\frac{1}{2\sigma_z^2} \frac{\left[\hat{Y}_{1,t} - h^{-1}(\bar{V})\right]^2}{nT-t} + \frac{1}{2}\log(nT-t) - \frac{\left[h^{-1}(\bar{V})\right]^2}{2\sigma_z^2}\right)}{1 - \pi_{nT-1} + \pi_{nT-1} \exp\left(\frac{1}{2\sigma_z^2} \frac{\left[\hat{Y}_{1,t} - h^{-1}(\bar{V})\right]^2}{nT-t} + \frac{1}{2}\log(nT-t) - \frac{\left[h^{-1}(\bar{V})\right]^2}{2\sigma_z^2}\right)}, \tag{C.28}$$

which is market makers' optimal estimate of the probability that the insider has private information, given the insider's order rate $\theta\left(t, \hat{Y}_{1,t}, V\right)$ defined by equation (C.20).

When the insider is not better informed, Lemma (9) implies that her trading strategy follows

$$\theta\left(t,y,\bar{V}\right) = -\frac{\Pi\left(t,y\right)\left[y-h^{-1}\left(\bar{V}\right)\right]}{nT-t} + \bar{\theta}\left(t,y\right),$$

which implies

$$\hat{\theta}\left(t\right) - \theta\left(t,y,\bar{V}\right) = \Theta\left(t,y\right) - \theta\left(t,y,\bar{V}\right) = \frac{y-h^{-1}\left(\bar{V}\right)}{nT-t}, \tag{C.29}$$

and

$$\Pi\left(t,y\right)\Theta\left(t,y\right) + \left[1-\Pi\left(t,y\right)\right]\theta\left(t,y,\bar{V}\right) = \bar{\theta}\left(t,y\right). \tag{C.30}$$

When the insider has no private information, I can rewrite the dynamics of the probability estimate as

$$\begin{aligned} d\Pi\left(t,\hat{Y}_{1,t}\right) = & \frac{\Pi\left(t,\hat{Y}_{1,t}\right)\left[1-\Pi\left(t,\hat{Y}_{1,t}\right)\right]}{\sigma_z^2}\frac{\hat{Y}_{1,t}-h^{-1}\left(\bar{V}\right)}{nT-t} \\ & \times\left(dZ_t - \frac{\Pi\left(t,\hat{Y}_{1,t}\right)\left[\hat{Y}_{1,t}-h^{-1}\left(\bar{V}\right)\right]}{nT-t}dt\right). \end{aligned} \tag{C.31}$$

Therefore, conditional on whether the insider is informed or not, there are two different dynamics of probability estimation, as stated in equations (C.26) and (C.31).

As stated in Lemma 10, a direct application of Theorem 1 in Li (2013) leads to the same property of probability estimate.

Lemma 10. *Let $\hat{Y}_{1,t}$ be the Brownian bridge as defined by equation (C.25) for any $V \in \mathbb{V}$. Suppose that the prior $\pi_{nT-1} \in (0,1)$. Then, market makers' probability estimate that the insider has private information, $\Pi\left(t,\hat{Y}_{1,t}\right)$, always resides in $(0,1)$ for all $t < nT$. Upon announcements, it converges to 1 or 0 depending on whether the insider has private information or not.*

Since $\log\left[H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)\right]$ is normally distributed with mean μ_v and variance $\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2$ at time $nT-1$, $h\left(y\right) = \exp\left(\mu_v + \frac{\beta-\gamma^A}{\beta}\lambda y\right)$ and $\bar{V} = e^{\mu_v+\frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2}$. From Lemma 9, $\forall t \in [nT-1, nT]$, the estimation of $H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)$ conditional on $\delta = 1$ follows

$$\begin{aligned} V\left(t\right) &= \mathbb{E}\left[H\left(\hat{m}_{nT},nT\right)A\left(\hat{m}_{nT},nT\right)|\mathcal{F}_{1,t}\right] \\ &= \mathbb{H}\left(t,\hat{Y}_{1,t}\right) = \mathbb{E}\left[h\left(\hat{Y}_{1,t}+Z_{nT}-Z_t\right)\right] \\ &= \exp\left(\mu_V + \frac{\beta-\gamma^A}{\beta}\lambda\hat{Y}_{1,t} + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2\left(nT-t\right)\right), \end{aligned}$$

while the estimation of $H(\hat{m}_{nT}, nT)\, A(\hat{m}_{nT}, nT)$ conditional on $\delta = 0$ is $\bar{V} = e^{\mu_V + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2}$, where $\mu_V = (\beta - \gamma^A)\, x_{nT-1} + N(nT)$.

Similarly, the estimation of SDF $H(\hat{m}_{nT}, nT)$ conditional on $\delta = 1$ follows

$$\begin{aligned}\Lambda(t) &= \mathbb{E}\left[H(\hat{m}_{nT}, nT) \,|\mathcal{F}_{1,t}\right] \\ &= \exp\left(\mu_\Lambda - \frac{\gamma^A}{\beta}\lambda \hat{Y}_{1,t} + \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (nT - t)\right),\end{aligned}$$

while the estimation of SDF $H(\hat{m}_{nT}, nT)$ conditional on $\delta = 0$ is $\bar{\Lambda} = e^{\mu_\Lambda + \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2}$, where $\mu_\Lambda = -\gamma^A x_{nT-1} + \mathcal{H}(nT)$.

Therefore, this implies the price defined in equation (33) depends only on the current adjusted trading flow $\hat{Y}_{1,t}$, which follows

$$\begin{aligned}P\left(t, \hat{Y}_{1,t}\right) &= \frac{\Pi\left(t, \hat{Y}_{1,t}\right) V\left(t, \hat{Y}_{1,t}\right) + \left(1 - \Pi\left(t, \hat{Y}_{1,t}\right)\right) \bar{V}}{\Pi\left(t, \hat{Y}_{1,t}\right) \Lambda\left(t, \hat{Y}_{1,t}\right) + \left(1 - \Pi\left(t, \hat{Y}_{1,t}\right)\right) \bar{\Lambda}} \\ &= \frac{\Pi\left(t, \hat{Y}_{1,t}\right) e^{\mu_V + \frac{\beta-\gamma^A}{\beta}\lambda \hat{Y}_{1,t} + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2 (nT-t)} + \left(1 - \Pi\left(t, \hat{Y}_{1,t}\right)\right) e^{\mu_V + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2}}{\Pi\left(t, \hat{Y}_{1,t}\right) e^{\mu_\Lambda - \frac{\gamma^A}{\beta}\lambda \hat{Y}_{1,t} + \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (nT-t)} + \left(1 - \Pi\left(t, \hat{Y}_{1,t}\right)\right) e^{\mu_\Lambda + \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2}} \\ &= P_{nT-1} \frac{\Pi\left(t, \hat{Y}_{1,t}\right) e^{\frac{\beta-\gamma^A}{\beta}\lambda \hat{Y}_{1,t} - \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2 (t-(nT-1))} + 1 - \Pi\left(t, \hat{Y}_{1,t}\right)}{\Pi\left(t, \hat{Y}_{1,t}\right) e^{-\frac{\gamma^A}{\beta}\lambda \hat{Y}_{1,t} - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (t-(nT-1))} + 1 - \Pi\left(t, \hat{Y}_{1,t}\right)},\end{aligned} \tag{C.32}$$

where $P_{nT-1} = e^{\beta \hat{m}_{nT-1} - \frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 + N(nT) + \frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2}$.

C.3 Step 2: Insider's Optimal Strategy

In this section, I show that if the dynamics of price follows equation (C.32), then the optimal trading strategy of the insider is indeed of the form given in equation (C.20) through verification proof.

Given market makers' pricing rule, $P(t) = P\left(t, \hat{Y}_{1,t}\right)$, the insider chooses the order rate to maximize her trading profit. When the insider has private information, for each terminal value $A(\hat{m}_{nT}, nT)$, she maximizes the terminal profit

$$\int_{nT-1}^{nT} \left(A(\hat{m}_{nT}, nT) - P\left(s, \hat{Y}_1(s)\right)\right) \theta_s ds.$$

When the insider is not bettered informed, given no new information coming before announcements, her best estimation of the asset value at $\forall t \in [nT-1, nT]$ is always

$$\bar{v}^* \equiv \mathbb{E}\left[\frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT|\mathcal{F}_{nT-1})]} A(\hat{m}_{nT}, nT)\,|\mathcal{F}^Y_{nT-1}\right]$$

$$= \frac{\mathbb{E}\left[H(\hat{m}_{nT}, nT)\, A(\hat{m}_{nT}, nT)\,|\mathcal{F}^Y_{nT-1}\right]}{\mathbb{E}\left[H(\hat{m}_{nT}, nT)\,|\mathcal{F}^Y_{nT-1}\right]} \equiv \frac{\bar{V}}{\bar{\Lambda}}, \tag{C.33}$$

which is the same as market makers at $t = nT-1$.

Under Assumption 1, the insider chooses the order rate to maximize the expectation of her terminal profit given the make makers' pricing rule $P(t) = P\left(t, \hat{Y}_{1,t}\right)$:

$$J(t, y; A(\hat{m}_{nT}, nT), \pi_{nT-1}) = \max_{\theta_t \in \mathcal{A}} \mathbb{E}\left[\int_t^{nT} \left(A(\hat{m}_{nT}, nT) - P\left(s, \hat{Y}_s\right)\right)\theta_s ds | \hat{Y}_{1,t} = y, A(\hat{m}_{nT}, nT)\right]$$

subject to

$$d\hat{Y}_t = \left[\theta(t) - \hat{\theta}(t)\right] dt + dZ_t, \tag{C.34}$$

where $A(\hat{m}_{nT}, nT) = \bar{v}^*$ when the insider is not informed as shown in equation (C.33).

The principle of optimality implies the following Bellman equation

$$\max_{\theta_t \in \mathcal{A}} \left\{ (A(\hat{m}_{nT}, nT) - P(t, y))\,\theta_t + J_t + J_y\left[\theta_t - \hat{\theta}(t)\right] + \frac{1}{2}\sigma_z^2 J_{yy} \right\} = 0 \tag{C.35}$$

where the subscripts denote the derivatives. The necessary conditions for having an optimal solution to the Bellman equation (C.35) are

$$J_y = P(t, y) - A(\hat{m}_{nT}, nT) \tag{C.36}$$

$$J_t + \frac{1}{2}\sigma_z^2 J_{yy} - \hat{\theta}(t)\, J_y = 0. \tag{C.37}$$

Under these necessary conditions, a direct application of Li (2013) leads to the following results:

Lemma 11. *Suppose the expected order rate $\hat{\theta}(t) = \Theta\left(t, \hat{Y}_{1,t}\right)$, where $\hat{Y}_{1,t}$ is the adjusted order at t. Let $\omega_t = y$ and suppose that the stochastic differential equation*

$$d\omega_s = dZ_s - \Theta(s, \omega_s)\, ds, \quad \forall nT \geq s \geq t \geq nT-1$$

has a unique solution, where Z_s is a Brownian motion with instant variance σ_z^2. If there exists a strictly monotone function $g(\cdot)$ such that the pricing rule is

$$P(t, y) = \mathbb{E}\left[g(\omega_{nT})\,|\omega_t = y\right], \tag{C.38}$$

then

$$J(t, y; v, \pi_{nT-1}) = \mathbb{E}\left[j(v, \omega_{nT})\,|\omega_t = y\right]$$

is a smooth solution to the Bellman equations (18) and (19), where

$$j(v, y) = \int_{y}^{g^{-1}(v)} [v - g(x)]\, dx \geq 0, \quad \forall (v, y)$$

Lemma 12. *Any continuous trading strategy that makes* $\lim_{t \to nT^-} P\left(t, \hat{Y}_{1,t}\right) = A\left(\hat{m}_{nT}, nT\right)$ *is optimal, where* $P(t, y)$ *is as defined by equation (C.38).*

Equation (C.38) implies that $P(t, \omega_t)$ is a martingale under the filtration generated by ω.[3] This implies the price dynamics with respect to $\mathcal{F}_{1,t}$ must satisfy

$$P_t - \Theta(t, y) P_y + \frac{1}{2}\sigma_z^2 P_{yy} = 0.$$

Finally, I am ready to prove that (X_0, X_1, P, Π) is an equilibrium. The insider's trading strategy, $X_{\delta,t}$, satisfies $X_0(t) = \int_{nT-1}^{t} \theta\left(s, \hat{Y}_1(s), \bar{V}\right) ds$ and $X_1(t) = \int_{nT-1}^{t} \theta\left(s, \hat{Y}_1(s), V\right) ds$, where $\hat{Y}_1$ is the solution to the stochastic differential equation (C.25). $\Pi(t, y)$ and $\theta(t, y, V)$ are defined by equations (C.28) and (C.20), respectively.

Proof of Theorem 3. Note that I have established that $\Pi\left(t, \hat{Y}_{1,t}\right)$ is the optimal probability estimate of market makers given the trading strategy in equation (C.20). Then, I need to show that the price dynamics in equation (C.32) is a legitimate pricing rule. That is,

[1]. The price rule defined above satisfies

$$P_t - \Theta(t, y) P_y + \frac{1}{2}\sigma_z^2 P_{yy} = 0, \tag{C.39}$$

[2]. $P\left(nT, \hat{Y}_{1,t}\right)$ is an increasing function of $\hat{Y}_{1,t}$; and

[3]. $\lim_{t \to nT} P\left(t, \hat{Y}_{1,t}\right) = A\left(\hat{m}_{nT}, nT\right)$ almost surely.

The first condition can be shown by direct calculation. For convenience, I let

$$P\left(t, \hat{Y}_{1,t}\right) \quad \equiv \quad P_{nT-1} \frac{A\left(t, \hat{Y}_{1,t}\right)}{B\left(t, \hat{Y}_{1,t}\right)}$$

[3]Notice that due to the existence of the SDF, the pricing rule $P(t)$ is no longer a martingale under market makers (unconditional) information set $\mathcal{F}_t^Y$. Though both $V(t)$ and $\Lambda(t)$ are martingales under $\mathcal{F}_t^Y$.

where

$$A\left(t,\hat{Y}_{1,t}\right)=\Pi\left(t,\hat{Y}_{1,t}\right)e^{\frac{\beta-\gamma^{A}}{\beta}\lambda\hat{Y}_{1,t}-\frac{1}{2}\left(\frac{\beta-\gamma^{A}}{\beta}\right)^{2}\sigma_{v}^{2}(t-(nT-1))}+1-\Pi\left(t,\hat{Y}_{1,t}\right),$$

and

$$B\left(t,\hat{Y}_{1,t}\right)=\Pi\left(t,\hat{Y}_{1,t}\right)e^{-\frac{\gamma^{A}}{\beta}\lambda\hat{Y}_{1,t}-\frac{1}{2}\left(\frac{\gamma^{A}}{\beta}\right)^{2}\sigma_{v}^{2}(t-(nT-1))}+1-\Pi\left(t,\hat{Y}_{1,t}\right).$$

In addition, I let $D\left(t,y\right)$ be defined as

$$D\left(t,\hat{Y}_{1,t}\right)=e^{\frac{\beta-\gamma^{A}}{\beta}\lambda\hat{Y}_{1,t}-\frac{1}{2}\left(\frac{\beta-\gamma^{A}}{\beta}\right)^{2}\sigma_{v}^{2}(t-(nT-1))}.$$

I also use the definition of $E\left(t,y\right)$ from equation (C.23).

The first-order conditions and second-order conditions of the price dynamics (C.32) are

$$P_{t}=P_{nT-1}B^{-2}\left\{\left[\left(\Pi_{t}-\frac{1}{2}\left(\frac{\beta-\gamma^{A}}{\beta}\right)^{2}\sigma_{v}^{2}\Pi\right)D-\Pi_{t}\right]B-A\left[\left(\Pi_{t}-\frac{1}{2}\left(\frac{\gamma^{A}}{\beta}\right)^{2}\sigma_{v}^{2}\Pi\right)E-\Pi_{t}\right]\right\},$$

$$P_{y}=P_{nT-1}B^{-2}\left\{\left[\left(\Pi_{y}+\frac{\beta-\gamma^{A}}{\beta}\lambda\Pi\right)D-\Pi_{y}\right]B-A\left[\left(\Pi_{y}-\frac{\gamma^{A}}{\beta}\lambda\Pi\right)E-\Pi_{y}\right]\right\},$$

and

$$P_{yy}=-P_{nT-1}B^{-2}\cdot 2B^{-1}\left\{\left[\left(\Pi_{y}+\frac{\beta-\gamma^{A}}{\beta}\lambda\Pi\right)D-\Pi_{y}\right]B-A\left[\left(\Pi_{y}-\frac{\gamma^{A}}{\beta}\lambda\Pi\right)E-\Pi_{y}\right]\right\}\left[\left(\Pi_{y}-\frac{\gamma^{A}}{\beta}\lambda\Pi\right)E-\Pi_{y}\right]+$$
$$P_{nT-1}B^{-2}\left\{\left[\left(\Pi_{yy}+2\frac{\beta-\gamma^{A}}{\beta}\lambda\Pi_{y}+\left(\frac{\beta-\gamma^{A}}{\beta}\right)^{2}\lambda^{2}\Pi\right)D-\Pi_{yy}\right]B-A\left[\left(\Pi_{yy}-2\frac{\gamma^{A}}{\beta}\lambda\Pi_{y}+\left(\frac{\gamma^{A}}{\beta}\right)^{2}\lambda^{2}\Pi\right)E-\Pi_{yy}\right]\right\}.$$

Put these derivatives into the following equation:

$$\frac{B^2}{P_{nT-1}}\left\{P_t-\left(\Theta-\bar{\theta}+\bar{\theta}\right)P_y+\frac{1}{2}\sigma_z^2P_{yy}\right\}$$

$$=\left\{\left[\left(\Pi_t-\frac{1}{2}\left(\frac{\beta-\gamma^A}{\beta}\right)^2\sigma_v^2\Pi\right)D-\Pi_t\right]B-A\left[\left(\Pi_t-\frac{1}{2}\left(\frac{\gamma^A}{\beta}\right)^2\sigma_v^2\Pi\right)E-\Pi_t\right]\right\}-$$

$$\left(\Theta-\bar{\theta}+\bar{\theta}\right)\left\{\left[\left(\Pi_y+\frac{\beta-\gamma^A}{\beta}\lambda\Pi\right)D-\Pi_y\right]B-A\left[\left(\Pi_y-\frac{\gamma^A}{\beta}\lambda\Pi\right)E-\Pi_y\right]\right\}+$$

$$\frac{1}{2}\sigma_z^2\left\{\left[\left(\Pi_{yy}+2\frac{\beta-\gamma^A}{\beta}\lambda\Pi_y+\left(\frac{\beta-\gamma^A}{\beta}\right)^2\lambda^2\Pi\right)D-\Pi_{yy}\right]B-A\left[\left(\Pi_{yy}-2\frac{\gamma^A}{\beta}\lambda\Pi_y+\left(\frac{\gamma^A}{\beta}\right)^2\lambda^2\Pi\right)E-\Pi_{yy}\right]\right\}$$

$$-B^{-1}\sigma_z^2\left\{\left[\left(\Pi_y+\frac{\beta-\gamma^A}{\beta}\lambda\Pi\right)D-\Pi_y\right]B-A\left[\left(\Pi_y-\frac{\gamma^A}{\beta}\lambda\Pi\right)E-\Pi_y\right]\right\}\left[\left(\Pi_y-\frac{\gamma^A}{\beta}\lambda\Pi\right)E-\Pi_y\right]$$

$$=\underbrace{\left[\Pi_t-\left(\Theta-\bar{\theta}\right)\Pi_y+\frac{1}{2}\sigma_z^2\Pi_{yy}\right]}_{\text{term (a)}}\left[B\left(D-1\right)-A\left(E-1\right)\right]+\underbrace{\left[\sigma_z^2\Pi_y-\left(\Theta-\bar{\theta}\right)\Pi\right]}_{\text{term (b)}}\left(\frac{\beta-\gamma^A}{\beta}\lambda DB+\frac{\gamma^A}{\beta}\lambda AE\right)$$

$$-\underbrace{\left(\bar{\theta}+B^{-1}\sigma_z^2\left(\left(\Pi_y-\frac{\gamma^A}{\beta}\lambda\Pi\right)E-\Pi_y\right)\right)}_{\text{term (c)}}\left\{\left[\left(\Pi_y+\frac{\beta-\gamma^A}{\beta}\lambda\Pi\right)D-\Pi_y\right]B-A\left[\left(\Pi_y-\frac{\gamma^A}{\beta}\lambda\Pi\right)E-\Pi_y\right]\right\}. \tag{C.40}$$

Next, I will show all of term (a), (b), and (c) are zero under the trading strategy (C.20). From equations (C.26) and (C.27),

$$d\Pi\left(t,\hat{Y}_{1,t}\right)=\frac{\Pi\left(t,\hat{Y}_{1,t}\right)\left[1-\Pi\left(t,\hat{Y}_{1,t}\right)\right]}{\sigma_z^2}\frac{\hat{Y}_{1,t}-h^{-1}\left(\bar{V}\right)}{nT-t}d\hat{Y}\left(t\right),$$

which implies $\Pi\left(t,\hat{Y}_{1,t}\right)$ is a martingale under market makers' information set since $\hat{Y}\left(t\right)$ is a Brownian motion under $\mathcal{F}_t^Y$. In addition,

$$\begin{aligned} d\hat{Y}_{1,t} &= dY_t-\Theta\left(t,\hat{Y}_{1,t}\right)dt \\ &= \left[d\hat{Y}\left(t\right)+\bar{\theta}\left(t,\hat{Y}_{1,t}\right)dt\right]-\Theta\left(t,\hat{Y}_{1,t}\right)dt \\ &= d\hat{Y}\left(t\right)-\left[\Theta\left(t,\hat{Y}_{1,t}\right)-\bar{\theta}\left(t,\hat{Y}_{1,t}\right)\right]dt, \end{aligned} \tag{C.41}$$

I have

$$\Pi_t-\left[\Theta\left(t,y\right)-\bar{\theta}\left(t,y\right)\right]\Pi_y+\frac{1}{2}\sigma_z^2\Pi_{yy}=0 \tag{C.42}$$

This shows that term (a) in equation (C.40) is always zero for any $\left(t,\hat{Y}_{1,t}\right)$.

Moreover, from equation (C.28),

$$\Pi_y = \Pi \left(1 - \Pi\right) \frac{y - h^{-1}\left(\bar{V}\right)}{\sigma_z^2 \left(nT - t\right)}. \tag{C.43}$$

Combining with equation (C.21), I have

$$\sigma_z^2 \Pi_y - \left[\Theta\left(t, y\right) - \bar{\theta}\left(t, y\right)\right] \Pi = 0, \tag{C.44}$$

which implies term (b) in equation (C.40) is always zero for any $\left(t, \hat{Y}_{1,t}\right)$.

The definition of $\bar{\theta}\left(t, y\right)$ directly indicates term (c) in equation (C.40) is always zero for any $\left(t, \hat{Y}_{1,t}\right)$. Therefore, the price dynamics satisfy (C.39) in the condition [1], which holds for all states of nature.[4]

As $\Pi\left(nT, \hat{Y}_{1,nT}\right) = 1$ when the insider is better informed by Lemma 10, I have

$$P\left(nT, \hat{Y}_{1,t}\right) = P_{nT-1} e^{\lambda \hat{Y}_{1,t} - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 \left(t - \left(nT - 1\right)\right)},$$

which increases in $\hat{Y}_{1,t}$ since $\lambda > 0$. This verifies the condition [2].

From Lemma 10, when the insider is better informed,

$$\begin{aligned}
\lim_{t \to nT} \log P\left(t, \hat{Y}_{1,t}\right) &= \log P_{nT-1} - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 + \lambda \lim_{t \to nT} \hat{Y}_{1,t} \\
&= \log P_{nT-1} - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 + \beta \left(\hat{m}_{nT} - \hat{m}_{nT-1}\right) \quad \text{a.s.} \\
&= \log A\left(\hat{m}_{nT}, nT\right) \quad \text{a.s.}
\end{aligned}$$

The second equality holds since $\hat{Y}_{1,t}$ is a Brownian bridge that converges to $h^{-1}\left(V\right)$ almost surely. The third equality comes from the definition of P_{nT-1}. Therefore, the condition [3] also holds.

■

Proof of Proposition 2. When the insider is informed, from Lemma 10 and Theorem 3, I show $P\left(t, \hat{Y}_{1,t}\right)$ converges to the true asset value $A\left(\hat{m}_{nT}, nT\right)$ almost surely when $t \to nT$. Since all the private information is eventually incorporated into the price, it implies there is no uncertainty left just upon announcements:

$$Var\left[\log P_{nT^-} | \mathcal{F}_{nT^-}^Y\right] = 0. \tag{C.45}$$

While when the insider is not informed, $P\left(t, \hat{Y}_{1,t}\right)$ converges to the initial price P_{nT-1} almost surely when $t \to nT$. There is no uncertainty reduction upon announcements since the insider has no information other

[4]Combining equations (C.43) and (C.22), I can derive $\bar{\theta}\left(t, y\right)$ as in equation (40).

than what market makers have at $nT-1$:

$$Var\left[\log P_{nT-}|\mathcal{F}^{Y}_{nT-}\right] = Var\left[\log P_{nT-}|\mathcal{F}^{Y}_{nT-1}\right] = \beta^2\Delta Q. \tag{C.46}$$

Therefore, when η fraction of insider that is informed across these FOMC announcements,

$$\log \mathbb{E}\left[\frac{P_{nT-}}{P_{nT-1}}\right] = \eta\gamma^{A}\beta\Delta Q, \tag{C.47}$$

$$\mathbb{E}\left[Var\left[\log P_{nT-}|\mathcal{F}^{Y}_{nT-}\right] - Var\left[\log P_{nT-}|\mathcal{F}^{Y}_{nT-1}\right]\right] = -\eta\beta^2\Delta Q, \tag{C.48}$$

where the expectations are taken over all states of natural. ■